Seminar 7: Scientific Visualization

Construct all numerical arrays from the analytic definitions below. No external datasets are required. Use MATLAB or an equivalent numerical language. All quantities are dimensionless unless otherwise stated. Distinguish the numerical model, sampling, and display settings. Label coordinates, colour scales, contour levels, seed locations, and glyph normalizations. Retain the same scales in controlled comparisons.

Problem 1. Representations of an Analytic Field

On $\Omega = [-2\pi, 2\pi] \times [-\pi, \pi]$, set $\varepsilon = 0.35$ and define

$$D(x, y) = \cosh y + \varepsilon \cos x, \qquad A_z = \ln D, \tag{1}$$

$$B_x = \frac{\sinh y}{D}, \qquad B_y = \frac{\varepsilon \sin x}{D}, \tag{2}$$

$$j_z = -\frac{1 - \varepsilon^2}{D^2}, \qquad B_z = 0. \tag{3}$$

Use $B_0 = a = \mu_0 = 1$, 161 points in x, and 81 points in y. For MATLAB's `meshgrid` convention, rows correspond to y and columns to x.

Tasks:

1. Construct maps of A_z and $|j_z|$, labelled contours, and a surface plot. State what the plotting height, colour, and contour levels represent.
2. Display the same magnetic field with sparse arrows and seeded field lines. Retain the coordinates and aspect ratio.
3. Combine the current-density map, magnetic-flux contours, and field lines in one annotated figure. Explain the information supplied by each layer.

Questions: Which information is omitted by each representation? Under what relation between A_z and $\mathbf{B}$ do the flux contours trace field lines?

Submission: One comparison figure and the script that constructs the arrays and renders the views.

Problem 2. Magnetic Reversal and Line Profiles

On $[-2, 2]^2$, use a uniform 301×301 grid and define

$$B_x = \tanh(y/a), \qquad B_y = 0.25, \qquad j_z = -\frac{1}{a}\operatorname{sech}^2(y/a), \qquad a = 0.30. \tag{4}$$

Use $B_0 = \mu_0 = 1$ and retain the fixed current reference $J_{\text{ref}} = 1/0.30$ in all comparisons.
Tasks:

1. Plot B_x with symmetric colour limits. Superpose magnetic-field lines on a map of $|j_z|$ and identify the scalar represented by the colourbar.
2. Extract the current profile at $x = 0$. Estimate its full width at half maximum by interpolating the two half-maximum crossings.
3. Repeat the calculation for $a = 0.15$. Keep the current reference and comparison colour limits fixed rather than normalizing each run by its own peak.
4. Compare the measured peak currents and widths. Verify the analytic width

$$W = 2a \operatorname{arcosh} \sqrt{2}.$$

Questions: How can independent normalization conceal an amplitude change? Which parts of the figure describe the magnetic vector field and which describe its derived current density?
Submission: A composite map, the two profiles, and a table of measured values.

Problem 3. Flux Geometry and Grid Refinement

Use the analytic island field of Problem 1 with $\varepsilon = 0.35$. Construct the arrays independently on each of the grids

$$N_x = 81,\ 161,\ 321,\ 641, \qquad N_y = (N_x + 1)/2. \tag{5}$$

The domain remains $[-2\pi, 2\pi] \times [-\pi, \pi]$, so that the physical sampling intervals in x and y are equal. At the X-point $(0, 0)$ and O-point $(\pi, 0)$,

$$A_X = \ln(1 + \varepsilon), \qquad A_O = \ln(1 - \varepsilon), \qquad \Delta\Psi = |A_X - A_O|. \tag{6}$$

Tasks:

1. Plot the current magnitude and flux contours, identifying the X-point, O-point, and separatrix $A_z = A_X$.
2. Measure the transverse island width on $x = \pi$ and the flux difference $\Delta\Psi$ on all four grids. State the crossing-interpolation rule.
3. Calculate $j_z = \partial_x B_y - \partial_y B_x$ by numerical differentiation and compare it with the analytic expression. Plot the relative peak-current error.
4. Compare the convergence of the width, flux difference, and peak current. The reference width is $W = 2\,\mathrm{arcosh}(1 + 2\varepsilon)$.

Questions: Why is $\Delta\Psi$ unchanged by adding a constant to A_z? Why does a static X-point not determine a reconnection rate?
Submission: One annotated figure and a grid-refinement table or graph.

Problem 4. Multiscale Sampling and Interpolation

On $[-2,2]^2$, define

$$
\begin{aligned}
g_1 &= \exp\left[-\left(\frac{y-0.38\sin(2.2x)-0.15\cos(3.5x)}{0.16}\right)^2\right],\\
g_2 &= \exp\left[-\left(\frac{y+0.85+0.18\sin(3.1x)}{0.23}\right)^2\right],\\
q &= \sqrt{\left(\frac{x+0.9}{0.72}\right)^2+\left(\frac{y-0.9}{0.58}\right)^2}, \qquad g_3 = \exp\left[-\left(\frac{q-1}{0.23}\right)^2\right],\\
g_4 &= \exp\left[-\left(\frac{x-1.05}{0.38}\right)^2-\left(\frac{y-0.95}{0.43}\right)^2\right],\\
g_5 &= [1+\sin(4x+0.9y)]\exp[-0.6(x^2+y^2)],\\
f(x,y) &= 0.08+0.65g_1+0.47g_2+0.88g_3+0.72g_4+0.20g_5.
\end{aligned}
$$

Sample f on a 29×29 grid. Independently evaluate the same analytic function on a 401×401 reference grid. Divide all arrays by the *same* coarse-grid maximum.

Tasks:

1. Compare the original sampled display with linear and cubic interpolation onto the fine plotting grid. Use identical axes and colour limits.
2. Overlay common contour levels. Plot the interpolation errors relative to the independently evaluated analytic reference.
3. Report the root-mean-square error and any overshoot outside the range of the original samples. State the interpolation method and boundary treatment.
4. Identify features that are poorly resolved by the coarse grid and whose apparent structure depends strongly on the chosen interpolator.

Questions: Does a smooth display establish adequate model resolution? Why must the analytic reference not be substituted for the interpolated data?

Submission: One comparison figure, an error table, and a reproducible script.

Problem 5. Nested Surfaces and Visibility

For $0 \le u, v \le 2\pi$, define three closed toroidal surfaces by

$$x = (R + r\cos v)\cos u, \qquad y = (R + r\cos v)\sin u, \qquad z = r\sin v, \tag{7}$$

where $R = 2.1$ and $r \in \{0.90, 0.59, 0.28\}$. Use 241 values of u and 121 values of v, including the coincident periodic endpoints.

Tasks:

1. Construct all coordinates analytically. Render the three surfaces with one geometric scale and fixed categorical colours.
2. Compare opaque and transparent outer shells using the same camera, lights, and geometry. State the opacity assigned to each surface.
3. Vary only one parameter at a time: outer-shell opacity, light direction, and camera orientation. Identify which interior surfaces are visible.
4. Export a turntable movie containing 192 frames at 24 frames per second. Keep the surfaces, opacity, colour mapping, and figure dimensions fixed.

Questions: How does a closed graphical surface differ from a solid material boundary? Why is display opacity not automatically a physical optical depth?

Submission: A labelled comparison, an eight-second movie, and the rendering script.

Problem 6. Vector Encodings and Field Reversal

Construct the analytic island field and uniform grid from Problem 1. Use the same arrays in every representation. The input noise texture used for LIC is a graphical construction, not a plasma variable.

Tasks:

1. Compare magnitude-scaled arrows, seeded field lines, and line integral convolution (LIC). State what each visual channel represents.
2. Repeat the arrow display with normalized vectors. Mask any points where the vector magnitude vanishes.
3. Change arrow spacing, streamline seed locations, and the LIC kernel length separately. Explain how each change alters the representation.
4. Reverse the magnetic field while keeping the input noise, integration settings, and graphical scales fixed. Compare the resulting images.
5. Test whether the numerically traced field lines are tangent to $\mathbf{B}$ and follow contours of A_z.

Questions: Which encodings preserve vector direction, which preserve magnitude, and which preserve only orientation? Why is symmetric LIC insensitive to $\mathbf{B} \mapsto -\mathbf{B}$?

Implementation: The supplied `lic2.m` is a source-code helper, not a MATLAB built-in. Alternatively, implement symmetric texture integration using bilinear sampling on the uniform grid. Use a fixed random seed for controlled comparisons.

Submission: One three-panel figure, one controlled parameter comparison, and an interpretation.

Problem 7. Tensor Glyphs and Rotation

At the 81 points of a uniform 9×9 grid on $[-2, 2]^2$, define

$$\theta = 0.5x + 0.25y, \qquad \phi = 0.18 \sin x, \qquad s = 1 + 0.25 \cos x \cos y, \tag{8}$$

$$(\lambda_1, \lambda_2, \lambda_3) = s(0.4 + 0.1 \sin x,\ 1,\ 1.6 - 0.1 \sin x), \tag{9}$$

$$\mathbf{P} = \mathbf{R}\ \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)\mathbf{R}^{\mathsf{T}}, \qquad \mathbf{R} = R_z(\theta)R_y(\phi). \tag{10}$$

Here R_z and R_y are the standard active rotations,

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad R_y(\phi) = \begin{pmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{pmatrix}. \tag{11}$$

Use $p_0 = 1$ and one common spatial glyph scale.

Tasks:

1. Construct the tensor arrays and verify symmetry and positive eigenvalues. Map a unit sphere through eigenvector rotation and semiaxis scaling $a_i = \sqrt{\lambda_i / p_0}$.
2. Compare the original tensors with the shape-normalized tensors $\mathbf{P}/(\mathrm{tr}\,\mathbf{P}/3)$. In the second view, encode the removed mean pressure using a shared scalar colour scale.
3. Animate the active rotation $\mathbf{P}(\alpha) = R_z(\alpha)\mathbf{P}R_z(\alpha)^{\mathsf{T}}$ through one full turn. Hold glyph centres, camera, scale, and colour limits fixed.
4. Check trace and eigenvalue invariance at several frames. Distinguish an arbitrary eigenvector sign from the sign of a principal value.

Questions: Which changes alter orientation and which alter intrinsic shape? What information is lost by independently normalizing every glyph?

Submission: Paired glyph fields, a video, and the positivity and invariance checks.