1 Physical Diagnostics from a Voigt Hα Profile

Suppose that the observed Hα line of a main-sequence star is well described by a Voigt profile. The Voigt function is the convolution of a Gaussian and a Lorentzian profile,

$$V(\nu) = G(\nu) * L(\nu). \tag{1}$$

If the Gaussian and Lorentzian parameters are measured, several simple physical quantities can be estimated directly from the fitted spectral shape.

The central wavelength of Hα is approximately:

$$\lambda_0 \simeq 6562.8 \text{ Å}. \tag{2}$$

1.1 Lorentzian width and coherence time

Consider an exponentially decaying temporal coherence function,

$$C_L(\tau) = C_0 e^{-|\tau|/\tau_c} e^{-i\omega_0\tau}. \tag{3}$$

$$L(\omega) \propto \frac{\gamma_\omega}{(\omega - \omega_0)^2 + \gamma_\omega^2}, \tag{4}$$

where

$$\gamma_\omega = \frac{1}{\tau_c}. \tag{5}$$

$$\gamma_\omega = 2\pi\gamma_\nu. \tag{6}$$

and

$$\tau_c = \frac{1}{2\pi\gamma_\nu}. \tag{7}$$

If Γ_L denotes the Lorentzian full width at half maximum,

$$\Gamma_L = 2\gamma_\nu, \tag{8}$$

and therefore

$$\tau_c = \frac{1}{\pi\Gamma_L}. \tag{9}$$

Thus a broader Lorentzian component corresponds to a shorter characteristic coherence time.

For Hα, this is an effective coherence or dephasing time, because natural, collisional, pressure, and Stark broadening can contribute to the Lorentzian width.

$$\Delta\nu_{Stark} \propto n_e^{3/2} \tag{10}$$

Effect of electric microfields on H atom (plasma effect).

1.2 Gaussian width and characteristic velocity

For non-relativistic Doppler shifts, thermal width of the line (micro Dopler of thermal motions):

$$\frac{\Delta\nu}{\nu_0} \simeq \frac{v}{c}. \tag{11}$$

If σ_ν is the standard deviation of the Gaussian component,

$$\sigma_v = c\frac{\sigma_\nu}{\nu_0}. \tag{12}$$

In wavelength space,

$$\sigma_v \simeq c\frac{\sigma_\lambda}{\lambda_0}. \tag{13}$$

Thus the measured Gaussian width provides a characteristic line-of-sight velocity dispersion.
If the Gaussian broadening is purely thermal,

$$\sigma_v^2 = \frac{k_B T}{m_H}. \tag{14}$$

Therefore,

$$T = \frac{m_H}{k_B}\sigma_v^2. \tag{15}$$

Using the measured wavelength width,

$$T = \frac{m_H}{k_B}\left(c\frac{\sigma_\lambda}{\lambda_0}\right)^2. \tag{16}$$

This gives an effective Doppler temperature.
If microturbulent motions are also present,

$$\sigma_v^2 = \frac{k_B T}{m_H} + \xi^2, \tag{17}$$

where ξ is the microturbulent velocity.

2 Voigt profile

Temporal coherence function containing both exponential and Gaussian decorrelation:

$$C(\tau) = C_0 \exp\left(-i\omega_0\tau\right)\ \exp\left(-\frac{|\tau|}{\tau_L}\right)\ \exp\left(-\frac{\tau^2}{2\tau_G^2}\right) \tag{18}$$

The Fourier transform of a product is the convolution of the corresponding Fourier transforms.

$$V(\omega) = L(\omega) * G(\omega). \tag{19}$$

where convolutions is defined as

$$(L * G)(\omega) = \int_{-\infty}^{+\infty} L(\omega')\, G(\omega - \omega')\, d\omega'. \tag{20}$$

The Lorentzian coherence scale

$$\tau_L = \frac{1}{\gamma_\omega}, \tag{21}$$

while the Gaussian variation scale

$$\tau_G = \frac{1}{\sigma_\omega}. \tag{22}$$

Their ratio

$$\frac{\tau_L}{\tau_G} = \frac{\sigma_\omega}{\gamma_\omega}. \tag{23}$$

This ratio gives a simple measure of the relative importance of the two broadening components.

3 Conversion from wavelength to frequency

For a narrow spectral line

$$\nu = \frac{c}{\lambda} \tag{24}$$

A small wavelength variation

$$\Delta\nu \simeq -\frac{c}{\lambda_0^2}\Delta\lambda \tag{25}$$

Hence

$$\sigma_\nu = \frac{c}{\lambda_0^2}\sigma_\lambda \tag{26}$$

and

$$\gamma_\nu = \frac{c}{\lambda_0^2}\gamma_\lambda \tag{27}$$

The Lorentzian coherence time can therefore be written directly in terms of the wavelength at HWHM:

$$\tau_c = \frac{\lambda_0^2}{2\pi c\,\gamma_\lambda} \tag{28}$$

3.1 Terms

Half Width at Half Maximum - HWHM
Full Width at Half Maximum - FWHM

3.2 Voigt width

The Voigt profile

$$V(\omega) = L(\omega) * G(\omega), \tag{29}$$

and its half-maximum condition

$$V\left(\omega_0 \pm \frac{\Gamma_V}{2}\right) = \frac{1}{2}V(\omega_0), \tag{30}$$

cannot be solved analytically.

An accurate empirical/analytical approximation - Olivero–Longbothum approximation:

$$\Gamma_V \simeq 0.5346\,\Gamma_L + \sqrt{0.2166\,\Gamma_L^2 + \Gamma_G^2}. \tag{31}$$

The numerical coefficients were chosen to approximate the exact Voigt FWHM with high accuracy over the different Gaussian/Lorentzian mixtures.

Now this can be compared directly with the measured width of the Hα line.

3.3 Relative Gaussian and Lorentzian contribution

A useful dimensionless parameter

$$a = \frac{\gamma}{\sqrt{2}\sigma}. \tag{32}$$

For

$$a \ll 1, \tag{33}$$

the profile is mainly Gaussian, while for

$$a \gg 1, \tag{34}$$

the Lorentzian contribution becomes dominant.

a - relative importance of Gaussian and Lorentzian broadening.

4 Simple observational analysis

From the fitted Voigt parameters,

$$\sigma_\lambda, \qquad \gamma_\lambda, \tag{35}$$

we can calculate

$$\sigma_v = c\frac{\sigma_\lambda}{\lambda_0}, \tag{36}$$

$$T_{\text{Doppler}} = \frac{m_H}{k_B}\sigma_v^2, \tag{37}$$

$$\tau_c = \frac{\lambda_0^2}{2\pi c\,\gamma_\lambda}, \tag{38}$$

and

$$a = \frac{\gamma_\lambda}{\sqrt{2}\sigma_\lambda}. \tag{39}$$

These quantities allow a simple physical interpretation of the observed Hα profile in terms of velocity broadening, effective temperature, coherence time, and the relative importance of Gaussian and Lorentzian components.