Gaussian Broadening Budget for the Hα Line of HD 195965

1 Gaussian distribution and the characteristic time τ_G

Consider a Gaussian temporal coherence function,

$$C_G(\tau) = C_0 \exp\left(-\frac{\tau^2}{2\tau_G^2}\right). \tag{1}$$

The parameter τ_G is the characteristic time scale over which the correlation decreases. At $\tau = \tau_G$,

$$\frac{C_G(\tau_G)}{C_0} = e^{-1/2} \simeq 0.607. \tag{2}$$

The Fourier transform of a Gaussian is also a Gaussian. Therefore the corresponding spectral profile can be written as

$$G(\omega) \propto \exp\left[-\frac{(\omega-\omega_0)^2}{2\sigma_\omega^2}\right]. \tag{3}$$

Comparison of the time-domain and frequency-domain Gaussian forms gives

$$\sigma_\omega = \frac{1}{\tau_G}, \tag{4}$$

or

$$\tau_G = \frac{1}{\sigma_\omega}. \tag{5}$$

Thus a short Gaussian coherence time corresponds to a broad Gaussian frequency distribution, while a long coherence time corresponds to a narrow frequency distribution.

2 From σ_ω to σ_ν

Angular frequency and ordinary frequency are related by

$$\omega = 2\pi\nu. \tag{6}$$

Therefore their Gaussian standard deviations satisfy

$$\sigma_\omega = 2\pi\sigma_\nu. \tag{7}$$

Hence

$$\tau_G = \frac{1}{2\pi\sigma_\nu}. \tag{8}$$

For a narrow spectral line,

$$\nu = \frac{c}{\lambda}, \tag{9}$$

and a small wavelength displacement gives

$$\Delta\nu \simeq -\frac{c}{\lambda_0^2}\Delta\lambda. \tag{10}$$

Therefore

$$\sigma_\nu = \frac{c}{\lambda_0^2}\sigma_\lambda. \tag{11}$$

Combining these relations,

$$\tau_G = \frac{\lambda_0^2}{2\pi c\,\sigma_\lambda}. \tag{12}$$

For HD 195965, using

$$\lambda_0 = 6562.8\ \text{Å}, \qquad \sigma_\lambda = 2.80953\ \text{Å}, \tag{13}$$

we obtain

$$\sigma_\nu = 1.956 \times 10^{11}\ \text{Hz}, \tag{14}$$

$$\sigma_\omega = 1.229 \times 10^{12}\ \text{rad}\,\text{s}^{-1}, \tag{15}$$

and

$$\tau_G = 8.14 \times 10^{-13}\ \text{s}. \tag{16}$$

3 The same Gaussian width in wavelength, frequency, and velocity

The Gaussian standard deviation can be expressed in three equivalent ways:

$$\sigma_\lambda, \qquad \sigma_\nu, \qquad \sigma_v. \tag{17}$$

For small Doppler shifts,

$$\frac{\Delta\lambda}{\lambda_0} \simeq \frac{v}{c}, \tag{18}$$

so

$$\sigma_v = c\frac{\sigma_\lambda}{\lambda_0}. \tag{19}$$

Equivalently,

$$\sigma_v = c\frac{\sigma_\nu}{\nu_0}. \tag{20}$$

For the measured Gaussian width,

$$\sigma_{v,\text{obs}} = 128.34 \pm 18.66\ \text{km}\,\text{s}^{-1}. \tag{21}$$

This quantity is a broadening scale. It is not the radial velocity of the star.

4 Gaussian broadening budget

In a simple second-moment description, independent approximately Gaussian broadening contributions add in quadrature:

$$\sigma_{v,\mathrm{obs}}^2 = \sigma_{v,\mathrm{th}}^2 + \sigma_{v,\mathrm{rot}}^2 + \sigma_{v,\mathrm{turb}}^2 + \sigma_{v,\mathrm{inst}}^2 + \sigma_{v,\mathrm{puls}}^2 + \sigma_{v,\mathrm{other}}^2. \tag{22}$$

After explicitly removing the thermal and rotational contributions, define the residual width

$$\sigma_{v,\mathrm{res}}^2 = \sigma_{v,\mathrm{obs}}^2 - \sigma_{v,\mathrm{th}}^2 - \sigma_{v,\mathrm{rot}}^2. \tag{23}$$

Therefore

$$\sigma_{v,\mathrm{res}}^2 = \sigma_{v,\mathrm{turb}}^2 + \sigma_{v,\mathrm{inst}}^2 + \sigma_{v,\mathrm{puls}}^2 + \sigma_{v,\mathrm{other}}^2. \tag{24}$$

This residual should not be identified with turbulence alone unless the other terms are independently known to be negligible.

5 Thermal broadening

For hydrogen atoms,

$$\sigma_{v,\mathrm{th}} = \sqrt{\frac{k_B T}{m_H}}. \tag{25}$$

Using

$$T = 26057 \text{ K}, \tag{26}$$

we obtain

$$\sigma_{v,\mathrm{th}} = 14.66 \text{ km s}^{-1}. \tag{27}$$

The equivalent wavelength width is

$$\sigma_{\lambda,\mathrm{th}} = \lambda_0 \frac{\sigma_{v,\mathrm{th}}}{c} = 0.321 \text{ Å}. \tag{28}$$

Thus thermal broadening is much smaller than the measured Gaussian width.

6 Rotational broadening

Rotation does not produce an exact Gaussian profile. However, for a simple broadening-budget estimate we can use the second moment of the standard rotational kernel with linear limb darkening ϵ:

$$\sigma_{v,\mathrm{rot}} = (v \sin i)\sqrt{\frac{15 - 7\epsilon}{20(3-\epsilon)}}. \tag{29}$$

Adopting

$$v \sin i = 240 \text{ km s}^{-1}, \qquad \epsilon = 0.4, \tag{30}$$

gives

$$\sqrt{\frac{15 - 7\epsilon}{20(3-\epsilon)}} = 0.4844, \tag{31}$$

and therefore

$$\sigma_{v,\mathrm{rot}} = 116.25 \text{ km s}^{-1}. \tag{32}$$

The corresponding equivalent wavelength second-moment width is

$$\sigma_{\lambda,\mathrm{rot}} = \lambda_0 \frac{\sigma_{v,\mathrm{rot}}}{c} = 2.545 \text{ Å}. \tag{33}$$

7 Residual Gaussian width

Using the observed, thermal, and rotational widths,

$$\sigma_{v,\mathrm{res}} = \sqrt{(128.34)^2 - (14.66)^2 - (116.25)^2}, \tag{34}$$

we obtain

$$\sigma_{v,\mathrm{res}} = 52.37 \ \mathrm{km\,s^{-1}}. \tag{35}$$

In wavelength units,

$$\sigma_{\lambda,\mathrm{res}} = \lambda_0 \frac{\sigma_{v,\mathrm{res}}}{c} = 1.146 \ \text{Å}. \tag{36}$$

If this residual width is represented by an effective Gaussian spectral profile, its frequency width is

$$\sigma_{\nu,\mathrm{res}} = 7.98 \times 10^{10} \ \mathrm{Hz}, \tag{37}$$

and its corresponding Gaussian time scale is

$$\tau_{G,\mathrm{res}} = \frac{1}{2\pi\sigma_{\nu,\mathrm{res}}} = 1.99 \times 10^{-12} \ \mathrm{s}. \tag{38}$$

The residual time scale is longer because the residual Gaussian spectral component is narrower.

8 Possible sources of the residual width

8.1 Turbulent broadening

Random atmospheric velocity fields can broaden the line through Doppler shifts. In the simplest Gaussian description one may write

$$\sigma_{v,\mathrm{turb}} = \xi, \tag{39}$$

where ξ is an effective turbulent velocity dispersion.

If all remaining broadening were attributed to turbulence, the maximum effective value would be

$$\xi_{\max} \simeq \sigma_{v,\mathrm{res}} = 52.37 \ \mathrm{km\,s^{-1}}. \tag{40}$$

This is only an upper limit because instrumental, pulsational, and other effects may also contribute.

8.2 Instrumental broadening

A spectrograph with resolving power

$$R = \frac{\lambda}{\Delta\lambda} \tag{41}$$

has a finite instrumental width. If the instrumental line-spread function is approximately Gaussian,

$$\mathrm{FWHM}_{\mathrm{inst}} \simeq \frac{\lambda_0}{R}, \tag{42}$$

and therefore

$$\sigma_{\lambda,\mathrm{inst}} \simeq \frac{\lambda_0}{2.355\,R}. \tag{43}$$

In velocity units,

$$\sigma_{v,\mathrm{inst}} \simeq \frac{c}{2.355\,R}. \tag{44}$$

The resolving power of the observational spectrum is required before this contribution can be evaluated and removed.

8.3 Pulsational broadening

A B-type star may exhibit pulsational surface velocity fields. Different surface elements can then contribute slightly different Doppler shifts. A time-integrated spectrum may therefore acquire an additional effective width,

$$\sigma_{v,\mathrm{puls}}. \tag{45}$$

Without a time-resolved line-profile analysis this contribution cannot be separated from the other residual velocity terms.

8.4 Other possible contributions

Other contributions may include macroturbulent atmospheric motions, unresolved multiple velocity components, imperfect continuum normalization, finite exposure-time effects, and deviations of the true rotational line profile from the simple Gaussian-equivalent second-moment treatment.

These effects are grouped schematically as

$$\sigma_{v,\mathrm{other}}. \tag{46}$$

9 Observed turbulence-related broadening in B stars

The residual width derived above can be compared with observational results for B-type stars. It is important to distinguish *microturbulence* from the additional line broadening conventionally parameterized as *macroturbulence*.

9.1 Microturbulence

Recent analyses of B-type dwarfs and giants commonly use very small microturbulent velocities. For example, Dufton et al. (2025) fixed the microturbulent velocity at

$$\xi_{\mathrm{micro}} = 2\ \mathrm{km\,s^{-1}}, \tag{47}$$

noting that there is little evidence that B-type dwarfs and giants have large microturbulence. Earlier work also finds microturbulent velocities of only a few $\mathrm{km\,s^{-1}}$ for many B stars, although larger values can occur in more massive or evolved objects.

For comparison with HD 195965,

$$\xi_{\mathrm{micro}} \sim 2\ \mathrm{km\,s^{-1}} \ll \sigma_{v,\mathrm{res}} = 52.37\ \mathrm{km\,s^{-1}}. \tag{48}$$

Therefore the residual width cannot naturally be identified with ordinary microturbulence alone.

9.2 Macroturbulent and pulsational broadening

Additional broadening of order tens of $\mathrm{km\,s^{-1}}$ is observed in B-type stars and is commonly parameterized as macroturbulence. Dufton et al. (2025) note explicitly that macroturbulent velocities in B-type dwarfs can be of order tens of $\mathrm{km\,s^{-1}}$.

Lefever et al. (2010), in a high-resolution spectroscopic study of 66 B-type stars, found substantial macroturbulent broadening in part of their sample. They also emphasize that a large fitted macroturbulent velocity can indicate time-dependent pulsational broadening. In this interpretation the "macroturbulent velocity" is a line-profile broadening parameter and should not automatically be interpreted as a literal isotropic turbulent gas speed.

The IACOB study of massive O- and B-type stars by Simón-Díaz et al. (2017) provides a broad observational framework for this additional macroturbulent line broadening and its connection with stellar parameters and pulsational phenomena.

For HD 195965 the present estimate,

$$\sigma_{v,\mathrm{res}} \simeq 52\ \mathrm{km\,s^{-1}}, \tag{49}$$

is therefore much larger than the characteristic microturbulent scale but is of the same general order as the additional macroturbulent/pulsational broadening encountered in early-type B-star spectroscopy.

If the entire residual were interpreted as a true turbulent velocity dispersion, it could be supersonic. However, the observational literature does not require this interpretation. A safer decomposition is

$$\sigma_{v,\mathrm{res}}^2 = \sigma_{v,\mathrm{micro}}^2 + \sigma_{v,\mathrm{macro}}^2 + \sigma_{v,\mathrm{inst}}^2 + \sigma_{v,\mathrm{puls}}^2 + \sigma_{v,\mathrm{other}}^2, \tag{50}$$

with

$$\sigma_{v,\mathrm{micro}} \sim \text{few}\ \mathrm{km\,s^{-1}} \tag{51}$$

and the larger residual potentially dominated by macroturbulent, pulsational, instrumental, or other large-scale broadening.

10 Numerical result

The broadening budget obtained from the current simple model is

$$\sigma_{v,\mathrm{obs}} = 128.34\ \mathrm{km\,s^{-1}}, \tag{52}$$

$$\sigma_{v,\mathrm{th}} = 14.66\ \mathrm{km\,s^{-1}}, \tag{53}$$

$$\sigma_{v,\mathrm{rot}} = 116.25\ \mathrm{km\,s^{-1}}, \tag{54}$$

and

$$\sigma_{v,\mathrm{res}} = 52.37\ \mathrm{km\,s^{-1}}. \tag{55}$$

Therefore

$$(52.37\ \mathrm{km\,s^{-1}})^2 = \sigma_{v,\mathrm{turb}}^2 + \sigma_{v,\mathrm{inst}}^2 + \sigma_{v,\mathrm{puls}}^2 + \sigma_{v,\mathrm{other}}^2. \tag{56}$$

The data therefore constrain the combined remaining Gaussian-like broadening to approximately

$$\sigma_{v,\mathrm{res}} \simeq 52\ \mathrm{km\,s^{-1}}, \tag{57}$$

but do not by themselves determine how this width is divided among turbulence, instrumental resolution, pulsations, and other effects.

11 Mean molecular weight for HD 195965

The stellar parameters used above give

$$T_{\mathrm{eff}} = 26057\ \mathrm{K}, \qquad [\mathrm{M/H}] \simeq +0.006, \tag{58}$$

so HD 195965 has approximately solar metallicity. The observational data do not directly provide the hydrogen and helium mass fractions X and Y. For a simple B0-star estimate we therefore adopt a near-solar composition,

$$X \simeq 0.70, \qquad Y \simeq 0.28, \qquad Z \simeq 0.02, \tag{59}$$

with

$$X + Y + Z = 1. \tag{60}$$

At $T \simeq 2.6 \times 10^4$ K the photospheric gas is strongly ionized, so for this simple estimate we treat hydrogen and helium as fully ionized.

For fully ionized hydrogen, one hydrogen nucleus contributes one proton and one electron. Hence the number of free particles per unit mass gives a contribution

$$\frac{2X}{m_H}. \tag{61}$$

For fully ionized helium, one helium nucleus of mass approximately $4m_H$ produces one helium ion and two electrons, i.e. three free particles. Therefore its contribution is

$$\frac{3Y}{4m_H}. \tag{62}$$

For the small metal fraction, a commonly used approximation is

$$\frac{Z}{2m_H}. \tag{63}$$

The total number of free particles per unit mass can therefore be written as

$$\frac{n}{\rho} = \frac{1}{m_H}\left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right). \tag{64}$$

The mean molecular weight μ is defined by

$$\rho = \mu m_H n, \tag{65}$$

so

$$\frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{1}{2}Z. \tag{66}$$

Using the adopted near-solar composition,

$$\frac{1}{\mu} = 2(0.70) + \frac{3}{4}(0.28) + \frac{1}{2}(0.02), \tag{67}$$

or

$$\frac{1}{\mu} = 1.40 + 0.21 + 0.01 = 1.62. \tag{68}$$

Hence

$$\mu \simeq 0.617 \simeq 0.62. \tag{69}$$

This is the value used in the sound-speed estimate,

$$c_s = \sqrt{\frac{\Gamma k_B T}{\mu m_H}}. \tag{70}$$

The value $\mu \simeq 0.62$ is therefore not measured directly from the spectrum; it follows from the approximately solar metallicity of HD 195965 together with the assumption of a nearly solar H/He mixture and strong ionization.

12 Mach number of the macroturbulent broadening

If the residual broadening is interpreted as a characteristic macroturbulent velocity dispersion, its Mach number can be estimated by comparing it with the sound speed.

For hydrogen thermal motions,

$$\sigma_{\rm th} = \sqrt{\frac{k_B T}{m_H}}. \tag{71}$$

The adiabatic sound speed of an ionized gas is

$$c_s = \sqrt{\frac{\Gamma k_B T}{\mu m_H}}, \tag{72}$$

where $\Gamma = 5/3$ and μ is the mean molecular weight.

Using the expression for $\sigma_{\rm th}$,

$$c_s = \sigma_{\rm th}\sqrt{\frac{\Gamma}{\mu}}. \tag{73}$$

Therefore the Mach number corresponding to the residual velocity scale is

$$\mathcal{M} = \frac{\sigma_{\rm res}}{c_s} = \frac{\sigma_{\rm res}}{\sigma_{\rm th}}\sqrt{\frac{\mu}{\Gamma}}. \tag{74}$$

For HD 195965,

$$\sigma_{\rm res} = 52.37\ {\rm km\,s^{-1}}, \qquad \sigma_{\rm th} = 14.66\ {\rm km\,s^{-1}}. \tag{75}$$

Taking

$$\mu \simeq 0.62, \qquad \Gamma = \frac{5}{3}, \tag{76}$$

gives

$$c_s \simeq 24.0\ {\rm km\,s^{-1}}, \tag{77}$$

and

$$\mathcal{M} \simeq \frac{52.37}{24.0} \simeq 2.2. \tag{78}$$

Thus, if the entire residual broadening were interpreted as a real macroturbulent velocity dispersion, it would correspond to a characteristic supersonic velocity scale with

$$\mathcal{M} \sim 2. \tag{79}$$

This value should be regarded as an upper-limit estimate because the residual width may also contain instrumental, pulsational, and other broadening contributions.

References

[1] Dufton, P. L., et al. (2025), "Characterization of B-type stars in four young Galactic open clusters – I. Stellar content and binary properties," *Monthly Notices of the Royal Astronomical Society*, **540**, 2009. The analysis fixes the microturbulent velocity of B-type dwarfs and giants at 2 $\mathrm{km\,s^{-1}}$ and notes that macroturbulence can be of order tens of $\mathrm{km\,s^{-1}}$ in B-type dwarfs.

[2] Lefever, K., Puls, J., Morel, T., Aerts, C., Decin, L., & Briquet, M. (2010), "Spectroscopic determination of the fundamental parameters of 66 B-type stars in the field-of-view of the CoRoT satellite," *Astronomy & Astrophysics*, **515**, A74, doi:10.1051/0004-6361/200911956.

[3] Simón-Díaz, S., Godart, M., Castro, N., et al. (2017), "The IACOB project III. New observational clues to understand macroturbulent broadening in massive O- and B-type stars," *Astronomy & Astrophysics*, **597**, A22.