LOCAL LINEAR STABILITY OF RHEOLOGICAL PROTOPLANETARY DISK FLOW WITH GRANULAR VISCOSITY

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Introduction

The linear stability of differentially rotating compressible protoplanetary disk flow, where dust and gas flow around the central gravitating object is studied. The effect of dust particle collisions is included using local rheological model (see e.g., Forterre 2008). In this framework the two component nature of the flow is ignored and one fluid description with rheological viscosity is adopted. Evolution of perturbations is studied in the context of planetesimal formation on early stages of planet formation in protoplanetary disks (Bodo *et al.* 2007, Tevzadze *et al.* 2008).

The axially symmetric formulation of the equilibrium disk flow is complemented with local rheological equation of state, when fluid viscosity depends on the pressure and velocity shear tensor. The radial viscous stress is balanced by large scale pressure gradient, while density gradient is set to lead isentropic flow profile.

The linear stability of small amplitude perturbations is studied by re-scaled perturbations according to the background disk properties. Local exponential instability due to viscous rheological forces is studied.

Protoplanetary Disk Flows

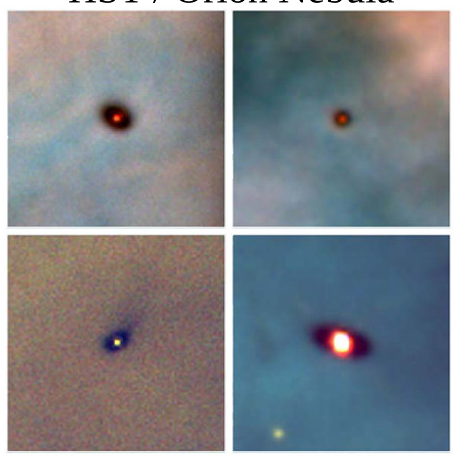
Protoplanetary disks are flows of gas and dust rotating around central object (protostar): *areas of planet formation.*

Keplerian rotation: $\Omega_K(r) = r^{-3/2}$

Observations

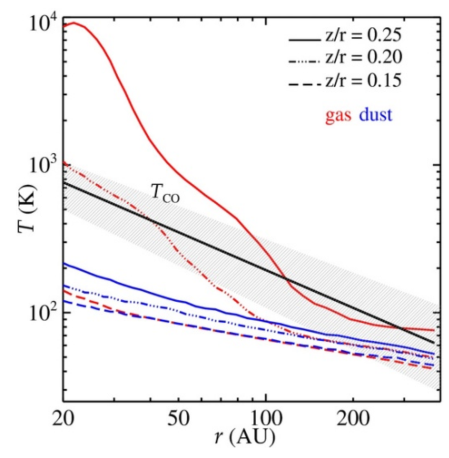
- Mixture of gas and micrometer sized dust granules
- Radial power low distribution of disk properties

Protoplanetary disks HST / Orion Nebula



μm dust granules Woitke et al. (2014)

Radial structure Fedele *et al.* 2013



Viscous Disk Model

Navier-Stoke s equation: $$\rho\left\{\frac{\partial}{\partial t} + v_k\frac{\partial}{\partial x_k}\right\}v_i = -\frac{\partial}{\partial x_i}P + \frac{\partial}{\partial x_k}\tau_{ik}$$

Viscous stress tensor: $$\tau_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i}\right)$$

Local rheological viscosity parameter: $\eta = \eta(P, \mathbf{V})$

Equilibrium disk model in polar coordinates:

$$\frac{V_{\phi 0}^2}{r} = \frac{1}{\rho_0}\frac{\partial P_0}{\partial r} + \frac{\partial \Phi}{\partial r} - \frac{1}{\rho_0}\left(\frac{\partial V_{\phi 0}}{\partial r} - \frac{V_{\phi 0}}{r}\right)\frac{\partial \eta}{\partial r}$$

$$\frac{1}{\rho_0}\frac{\partial \eta_0}{\partial r}\frac{\partial \Omega(r)}{\partial r} + \Omega(r)^2 = \frac{1}{\rho_0 r}\frac{\partial P_0}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial r}$$

Φ– gravitational potential of central object.

Keplerian disk solution – power law distrubution.
-Central gravity is balanced by centrifugal force;
-Pressure gradient is balanced by viscous term;

$$\rho_0 = \bar{\rho}\left(\frac{r}{r_0}\right)^{-\beta_\rho} \quad P_0(r) = \bar{P}\left(\frac{r}{r_0}\right)^{-\beta_p} \quad \eta(r) = \bar{\eta}\left(\frac{r}{r_0}\right)^{-\beta_\eta}$$

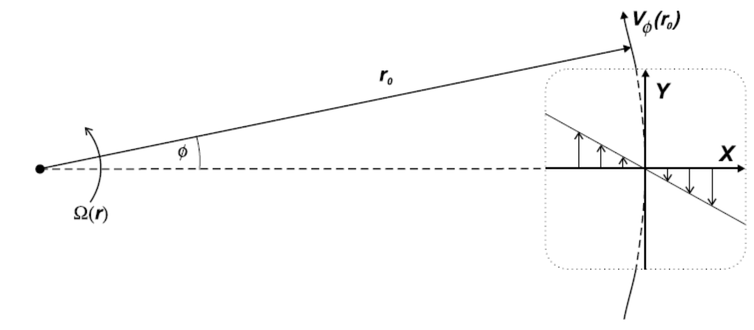
$$\beta_\Sigma = 2/\gamma \qquad \beta_\eta = 1/2 \qquad \beta_p = 2$$

Linear Perturbations

Radial power law scaling of linear perturbations:

$$\hat{\rho} = \left(\frac{r}{r_0}\right)^{-\delta_\Sigma}\rho' \qquad \hat{P} = \left(\frac{r}{r_0}\right)^{-\delta_p}P' \qquad \hat{V} = \left(\frac{r}{r_0}\right)^{-\delta_v}V'$$

Scaling factors are chosen later to remove complex terms in the dispersion equation due to background inhomogeneity.



Analysis in local Cartesian co-rotating shear sheet frame: Non-normal modes.

Spatial Fourier expansion in time: $$\begin{pmatrix}\hat{S}/\gamma\bar{P}\\ \hat{\mathbf{V}}\\ \hat{P}/\gamma\bar{P}\end{pmatrix} = \begin{pmatrix} i\varrho(\mathbf{k},t)\\ \mathbf{v}(\mathbf{k},t)\\ -ip(\mathbf{k},t)\end{pmatrix}\times\exp\left[ik_x x + ik_y y\right]$$

Linear stability using low frequency adiabatic approximation. $$\left|\frac{d}{dt}\omega(t)\right| \ll \omega^2(t)$$

Viscous Stability

General form of adiabatic dispersion equation for local linear perturbations in viscous Keplerian flow:

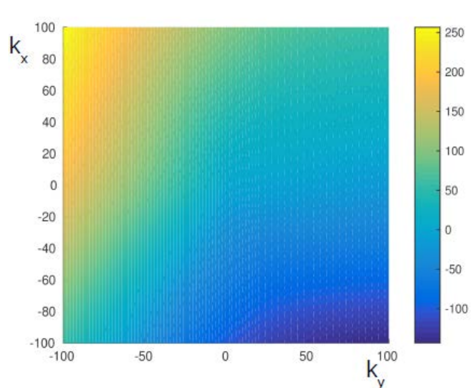
$$\begin{vmatrix} -i\omega & 0 & 0 & 0\\ -c_s^2 k_p - \nu A k_{\eta 1} & -i\omega + \nu\Gamma_1 & -2\Omega + \nu\zeta_1 & c_s^2(k_x + iO_1) + i\nu A k_{\eta 1}\\ \nu A k_{\eta 2} & -2B + \nu\zeta_2 & -i\omega + \nu\Gamma_2 & c_s^2 k_y + i\nu A k_{\eta 2}\\ 0 & -k_x - iO_2 & -k_y & -i\omega \end{vmatrix} = 0$$

Solutions: $$i\omega(\omega^2 - c_s^2 k^2 + 4B\Omega) + \nu D_1 + \nu^2 D_2 = 0$$

B - Oort's constant, c_s^2 - speed of sound.

Unstable mode: $$\omega = i\nu\frac{D_1}{c_s^2 k^2 + 4B\Omega}$$

Spectral distribution of the instability growth rate



Local linear rheological viscosity instability ~ *exp(σt)*
Instability of trailing (ky<0) modes.

Summary

Viscous rheological model of the gas-dust interaction in gravitating astrophysical disk flows is developed. Local linear exponential instability in Keplerian protoplanetary disks originating from the granular properties of dust particles is found.

Instability can lead to the growth of gas density and acceleration of dust particle agglomeration process, thus leading to rapid formation of planetesimals. Obtained results contribute to the core accretion model of the planet formation in protoplanetary disks.

References

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