Dynamics of Modes in Protoplanetary Discs with Radial Stratification

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ABSTRACT

We study the dynamics of perturbation modes in two dimensional compressible astrophysical discs with radial stratification and differential rotation using linear analysis and direct numerical simulations.

We extend the recent progress in the understanding of the baroclinic processes in protoplanetary discs and employ three-mode formalism for the description of the dynamics of the linear perturbations. In this framework we study the couplings of vorticity (W), entropy (S) and compressible (P) modes in the shearing sheet approximation. The system exhibits asymmetric three-mode coupling.

We show that in discs with sufficient radial equilibrium gradients energy transfer between the compressible, thermal and potential vorticity can occur kinematically due to the combined action of the differential rotation and radial stratification. Global 2D compressible numerical simulations of the semi-Keplerian disc dynamics show that compressible perturbations can generate aperiodic thermal perturbations on the shearing timescales. Described process can indicate a higher rate of the vortex production that will eventually facilitate the emergence of the long-lived vortical structures and acceleration of the planetesimal formation process.

Radially stratified equilibrium: sub-Keplerian flow

$$\bar{\Sigma}(r) = \Sigma_0 \left(\frac{r}{r_0}\right)^{-\beta_\Sigma}, \quad \bar{P}(r) = P_0 \left(\frac{r}{r_0}\right)^{-\beta_P}, \qquad \beta_S \equiv \beta_P - \gamma\beta_\Sigma .$$

Radial scaling of perturbation
in *global* frame

$$1 + \delta_V - \beta_P/\gamma = 0 ,$$
$$\delta_P + \beta_P/\gamma = 0 .$$

$$\hat{\Sigma}(r) \equiv \left(\frac{r}{r_0}\right)^{-\delta_\Sigma} \Sigma'(r) ,$$
$$\hat{P}(r) \equiv \left(\frac{r}{r_0}\right)^{-\delta_P} P'(r) ,$$
$$\hat{\mathbf{V}}(r) \equiv \left(\frac{r}{r_0}\right)^{-\delta_V} \mathbf{V}'(r) .$$

Adjusting ***global*** scaling for ***local*** purpose.

Analyzing the linear dynamics of spatial Fourier harmonics of perturbations in the shearing sheet framework. Radial scaling removes global trends and simplifies the local description.

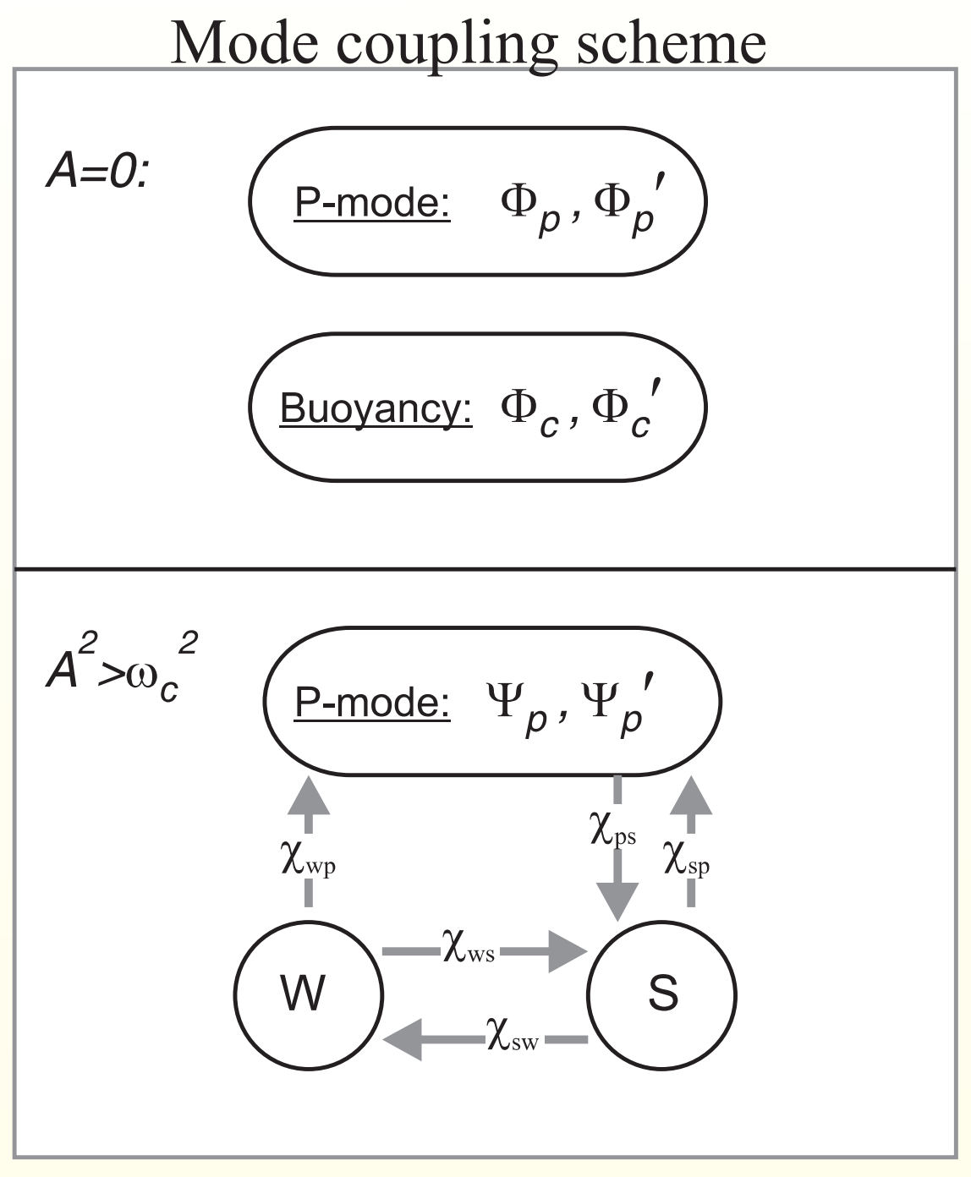
4th order linear system

A=0: **2** *2nd order modes*

$$\bar{\omega}_p^2 = c_s^2 k^2 + 4\Omega_0^2 ,$$
$$\bar{\omega}_c^2 = -\frac{c_s^4 \eta k_y^2}{c_s^2 k^2 + 4\Omega_0^2} .$$
$$\eta = \frac{\beta_P \beta_S}{\gamma^2 r_0^2} .$$

A=1/2r ∂/∂r log Ω(r): **1** *2nd order mode*
2 *1st order modes*

Mode coupling scheme



Time-scales in weakly stratified sub-Keplerian flow: $\bar{\omega}_c^2 \ll A^2 \ll \bar{\omega}_p^2$

3 mode formalism

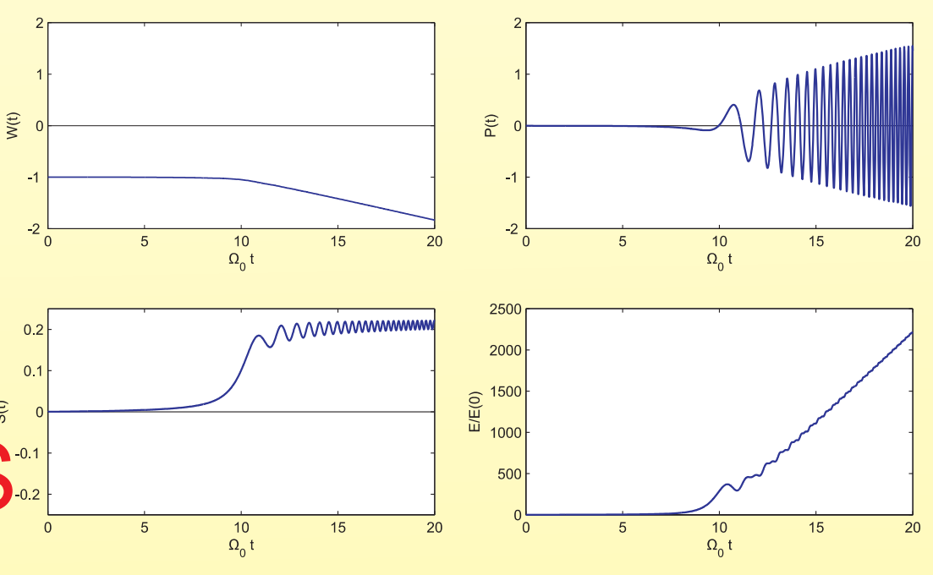
$$\left\{\frac{d^2}{dt^2} + f_p \frac{d}{dt} + \omega_p^2 - \Delta\omega_p^2\right\} \Psi_p = \chi_{pw} W + \chi_{ps} s ,$$
$$\left\{\frac{d}{dt} + f_s\right\} s = \chi_{sp1} \frac{d\Psi_p}{dt} + \chi_{sp2} \Psi_p + \chi_{sw} W ,$$
$$\frac{dW}{dt} = \chi_{ws} s ,$$

Ψ_p - eigenfunction of P mode, S - Entropy, W - Potential vorticity
ODE system describes the dynamics of 3 coupled shear modes: P, S and W.
Non-resonant coupling and mode conversion due to shear flow non-normality.

Linear Mode Transformations

Linear dynamics of perturbation SFH in time: initial perturbations correspond to one particular linear mode (P, S, W). Mode excitation occurs at times when radial wave-number changes sign: (kx(10)=0).

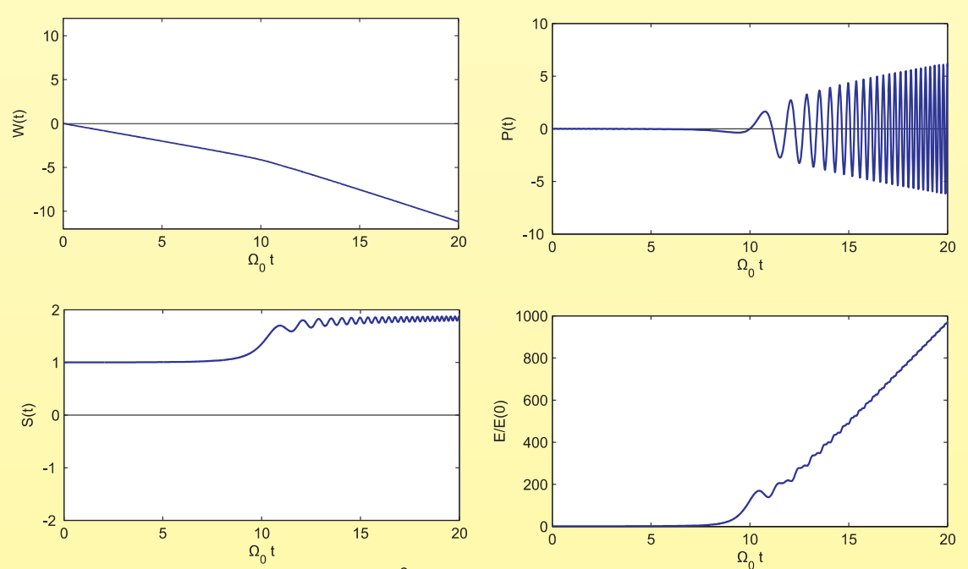
W ⟶ P, S



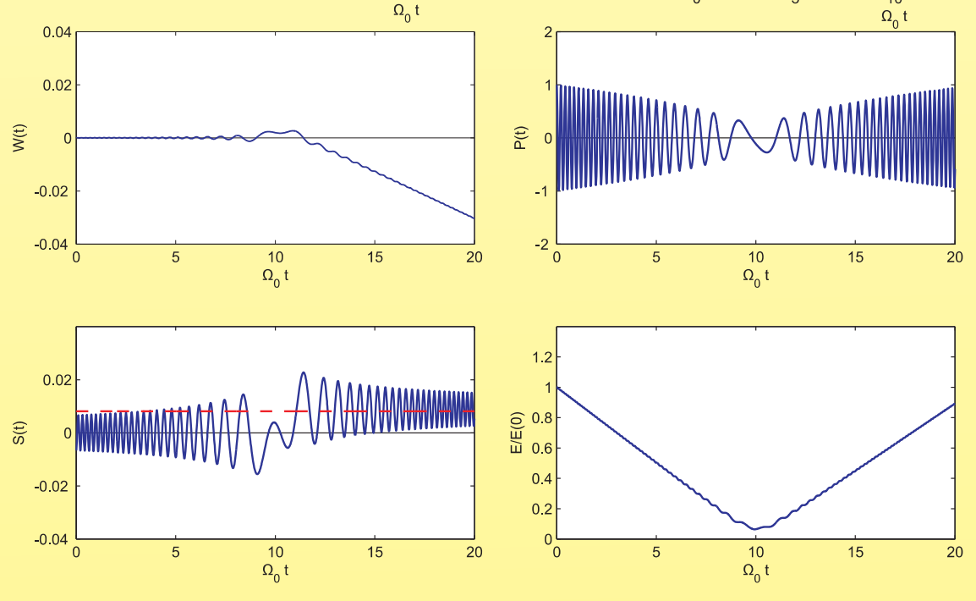
Mode Conversion Channel

$K_x = -10\ H^{-1}$
$K_y = 2\ H^{-1}$
$K_p = 0.2 H^{-1}$
$K_s = 0.2 H^{-1}$

S ⟶ P, W



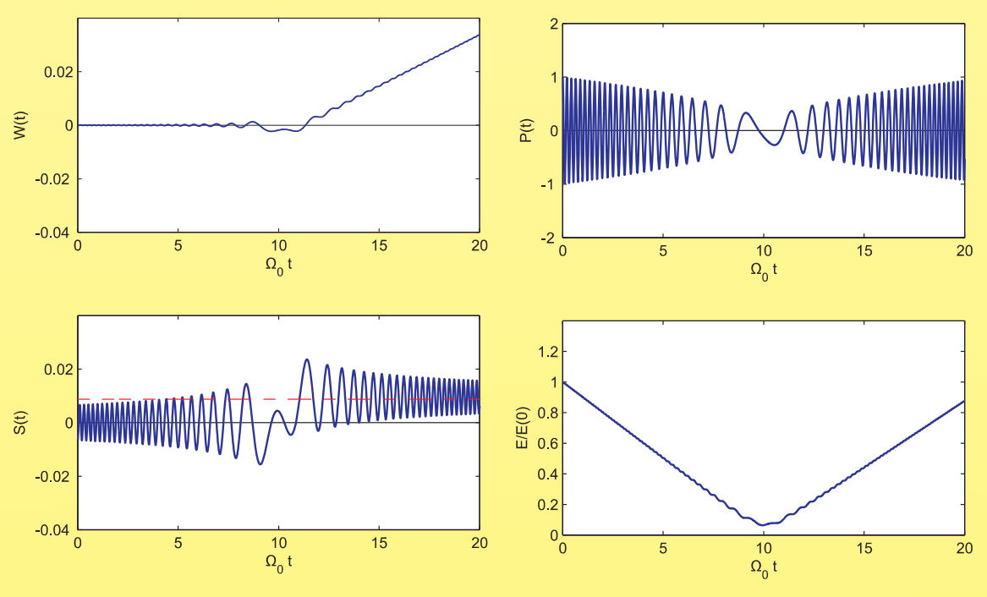
P ⟶ W, S



P ⟶ W, S

$K_x = -10\ H^{-1}$
$K_y = 2\ H^{-1}$
$K_p = 0.2 H^{-1}$
$K_s = -0.2 H^{-1}$

Baroclinically stable harmonics
excitation of potential vorticity



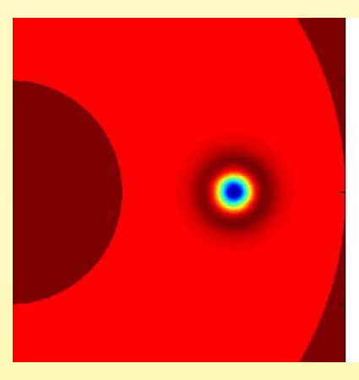
Numerical Simulations

PLUTO (plutocode.to.astro.it) **Riemann/Godunov, HD, FARGO, (ppm)**
Spatial interpolation: `linear, parabolic`
Time Stepping: `RGK2`
Solver: `two shock`

Coordinates: polar
Grid: (2048x326), (4096x652)

Equilirium:
Radially stratified sub-Keplerian flow;
beta_sigma = 1; beta_s = 0.8,

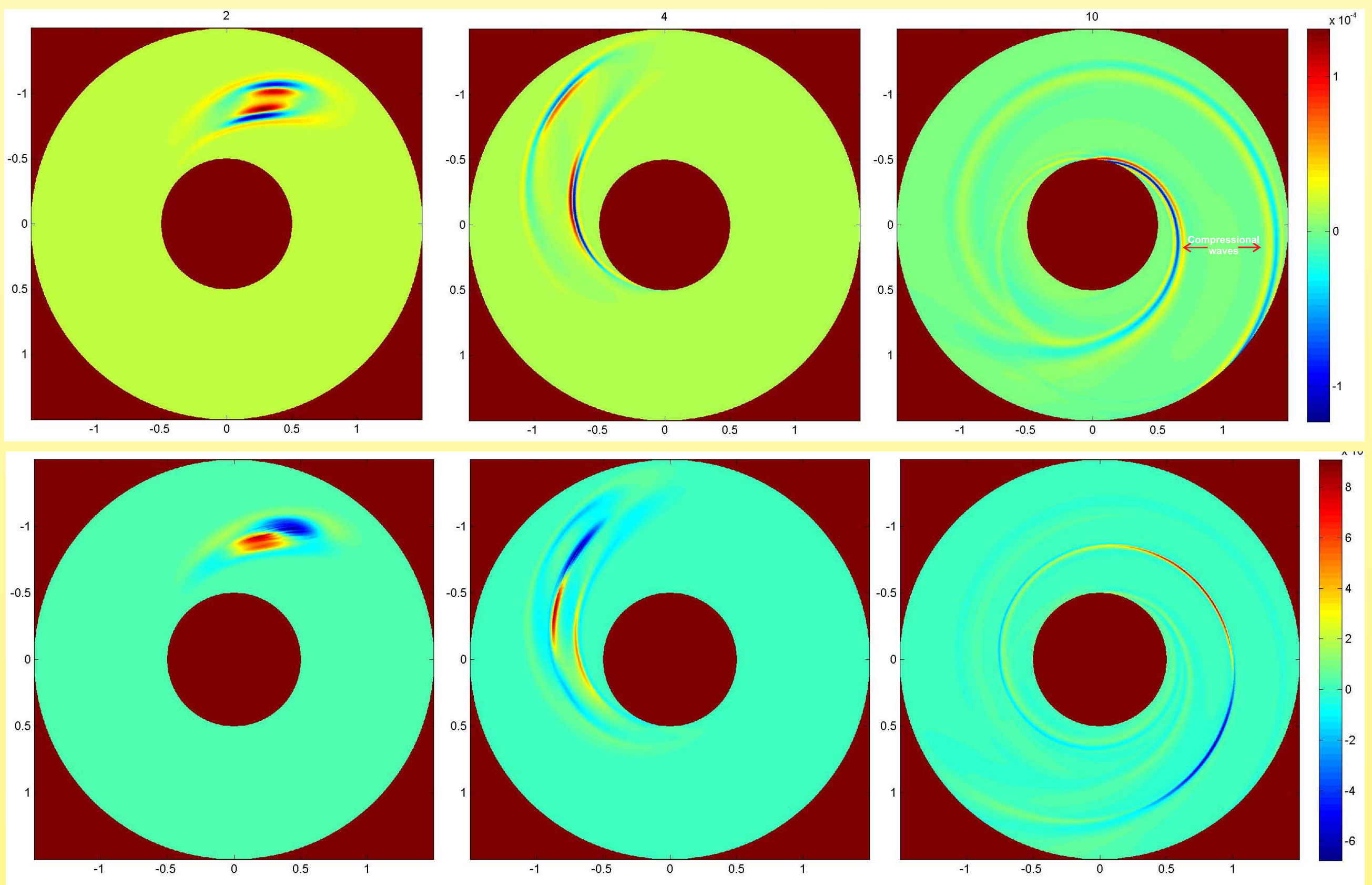
Initial Perturbation:
Compressible plume
pressure, density
zero entropy
zero P.Vorticity



small amplitudes perturbations

Pressure perturbation

Entropy perturbation

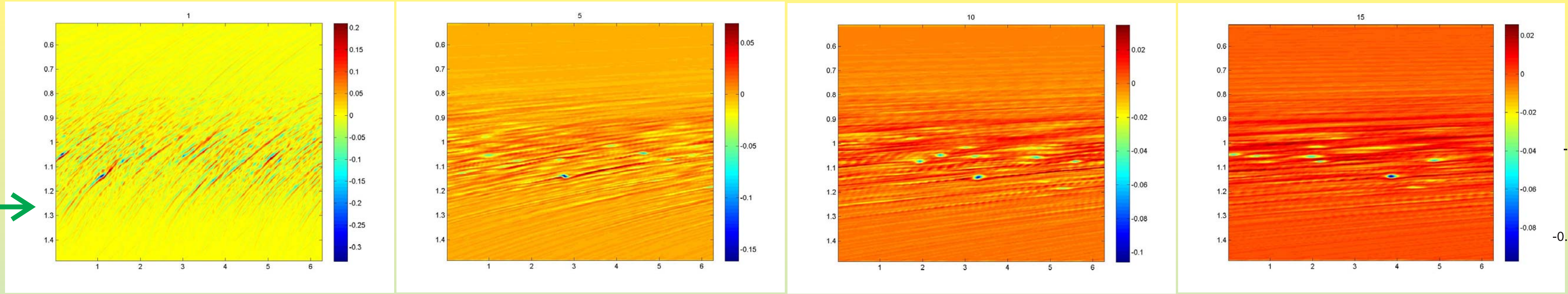


Initial perturbations correspond to compressible plume with zero entropy and potential vorticity perturbation over sub-Keplerian background. Positive and negative perturbations of entropy are excited due to the linear mode conversion phenomenon. t=0.1, 0.4 and 1 Ω^{-1} times are shown.

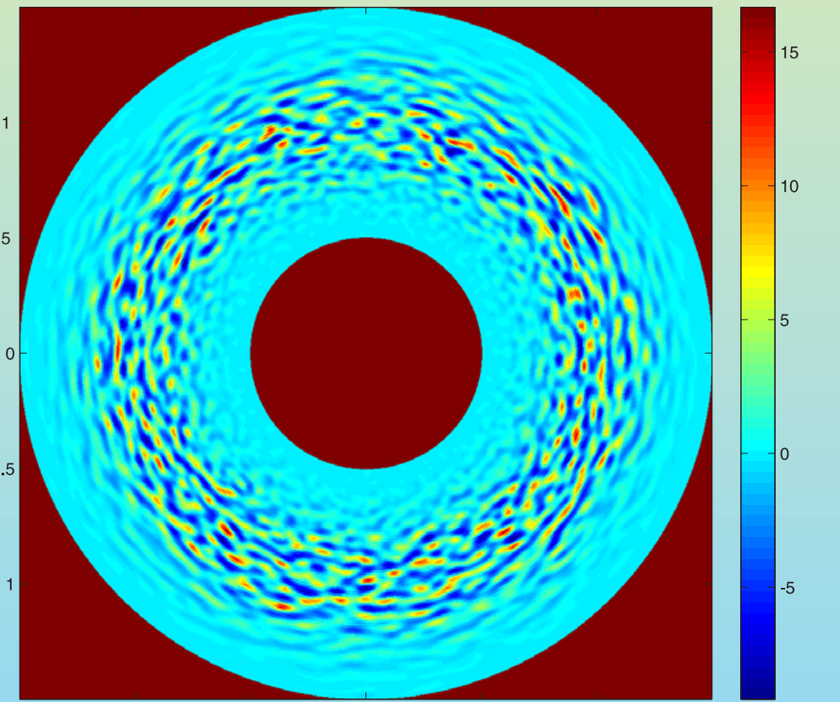
Random Compressible Perturbations

Development of potential vorticity of perturbations

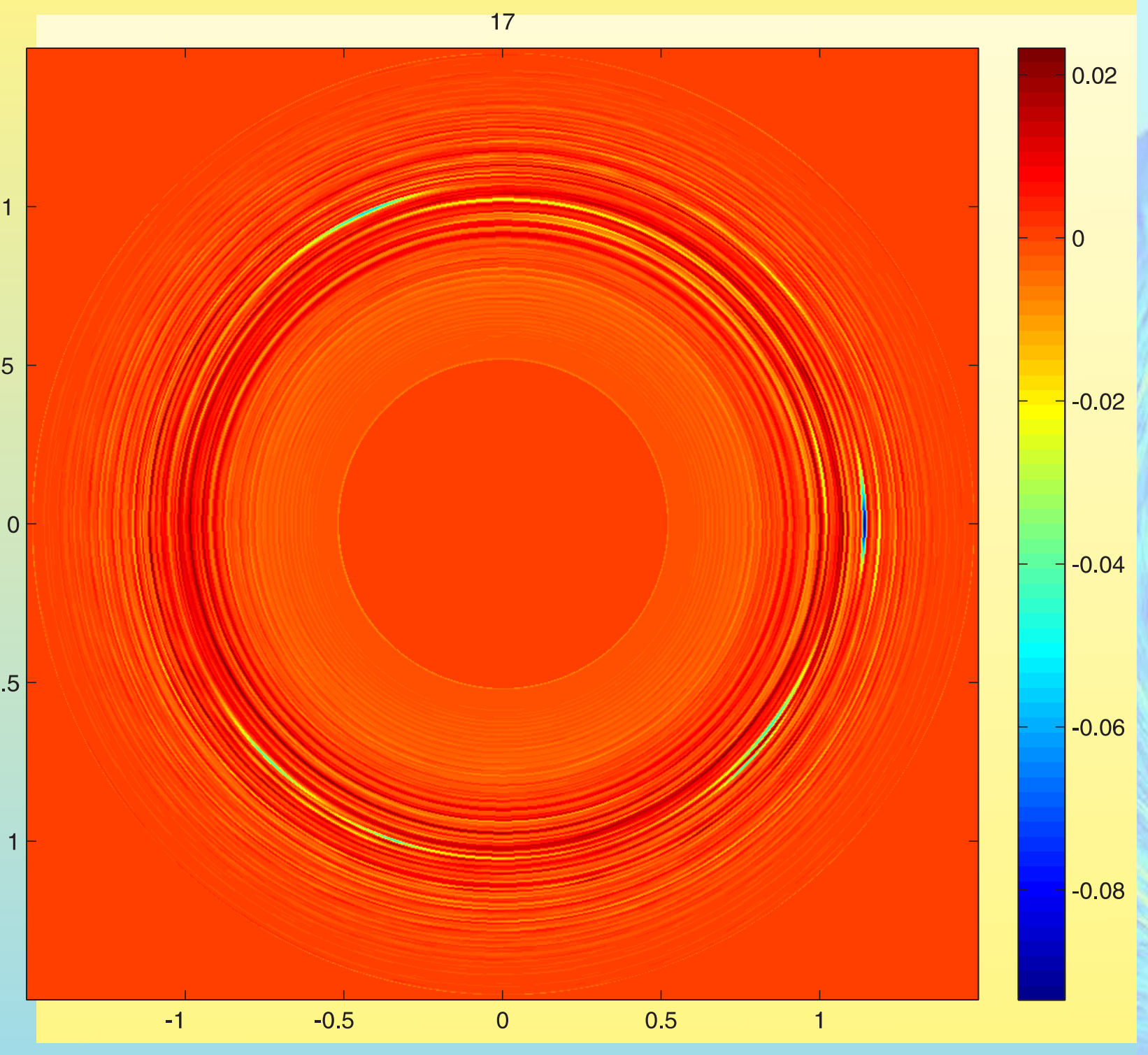
t = 1, 5, 15, 17; $\Omega(r=1)^{-1}$



Initial distribution of pressure perturbation



Development of anticyclonic vortical patches



Development of anticyclonic vortices

Sources of potential vorticity:

- Linear mode conversion mechanism
- Shocks: entropy production and potential vorticity generation

Competition at nonlinear amplitudes ...