Velocity shear induced phenomena in solar and astrophysical flows

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Outline

1. Introduction
2. General properties of shear flows
3. Linear mode conversion: excitation of acoustic waves
4. MHD shear flows: excitation of magnetosonic waves
5. Excitation of waves in convectively unstable shear flows
6. Dynamics of vortices in differentially rotating flows
7. Transformation of waves in MHD shear flows
8. Summary

➊ Introduction

Flows with velocity inhomogeneity widely occur in nature.

Shear flow analysis – long standing problem.

According to the classical Rayleigh's theory the existence of the inflection point in the velocity profile is necessary for the spectral instability.

A wide class of spectrally stable flows: <u>smooth shear flows</u>

Standard eigenmode analysis – modal approach fails to explain experimental data on shear flow dynamics.

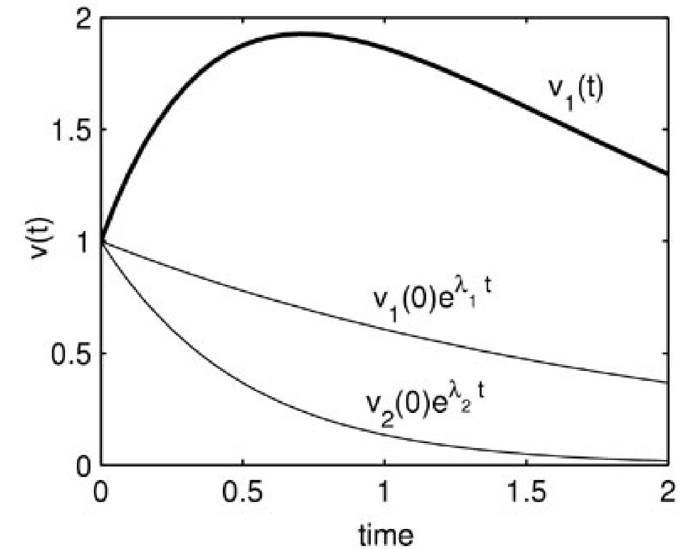
❶ Introduction

Shear flows are non-normal

Operators are not orthogonal - eigenfunctions interfere
Eigenmodes does not describe the system dynamics completely

Model non-normal system:

modal solutions decay, while the complete solution of the system exhibits the growth in a limited time interval.



Additional channels of energy exchange:

a) Background and perturbations;

b) Different perturbation modes;

❶ Introduction

Shear flow analysis: alternatives to the modal analysis

* Numerical study of the Orr-Sommerfeld equation;

* Partial integration of the equation of motion;

* Pseudospectrum;

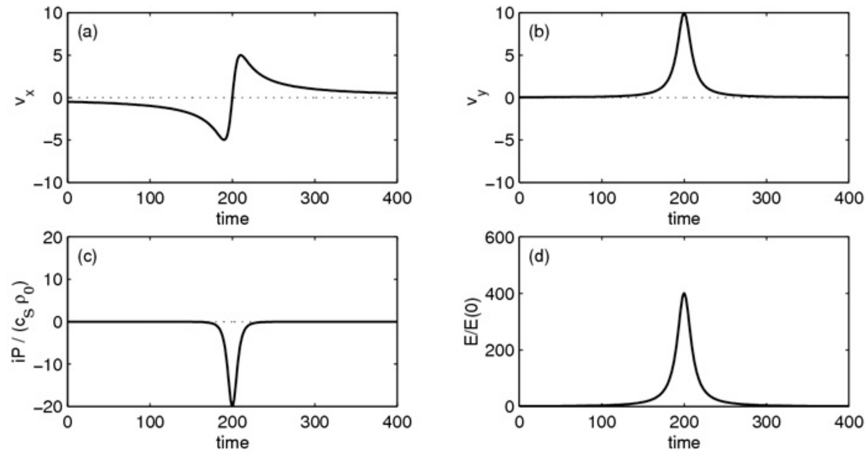
Most successful method in shear flow analysis:

Nonmodal approach

Shearing sheet transformation and consequent analysis of initial value problem in the wave-number space

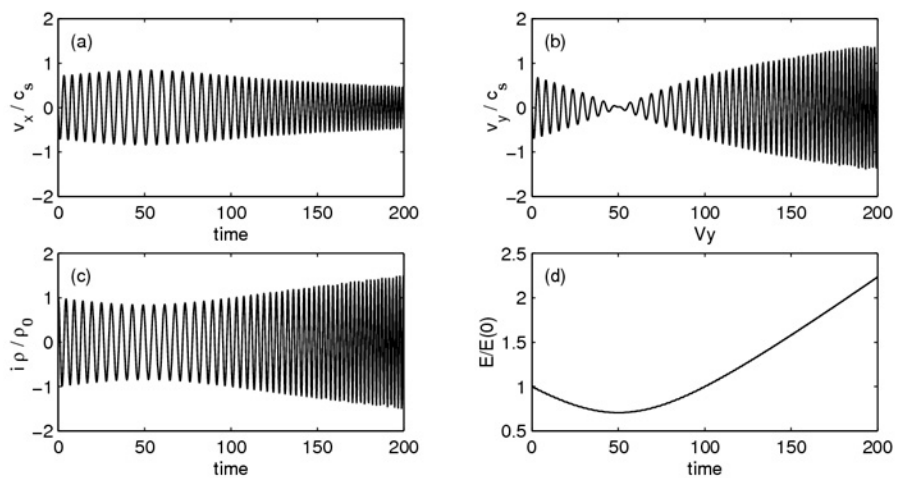
Nonmodal approach provides us the framework for present study

❷ General properties of shear flows



Two basic effects in shear flows:

Transient growth of vortical perturbations
wave number and frequency variation in time.



❸ Linear mode conversion

2D unbounded compressible parallel shear flow: **V =** ***(Ay,0)***

Shearless limit:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} v_x^{(\mathrm{w})}(t) + c_s^2 k^2 v_x^{(\mathrm{w})}(t) = 0$$

$$k^2 v_x^{(\mathrm{v})} = k_y I.$$

Velocity shear induced coupling

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} v_x(t) + c_s^2 k^2(t) v_x(t) = c_s^2 k_y(t) I,$$

Linear non-resonant mode conversion in shear flows:

Vortices excite acoustic wave modes

❸ Linear mode conversion

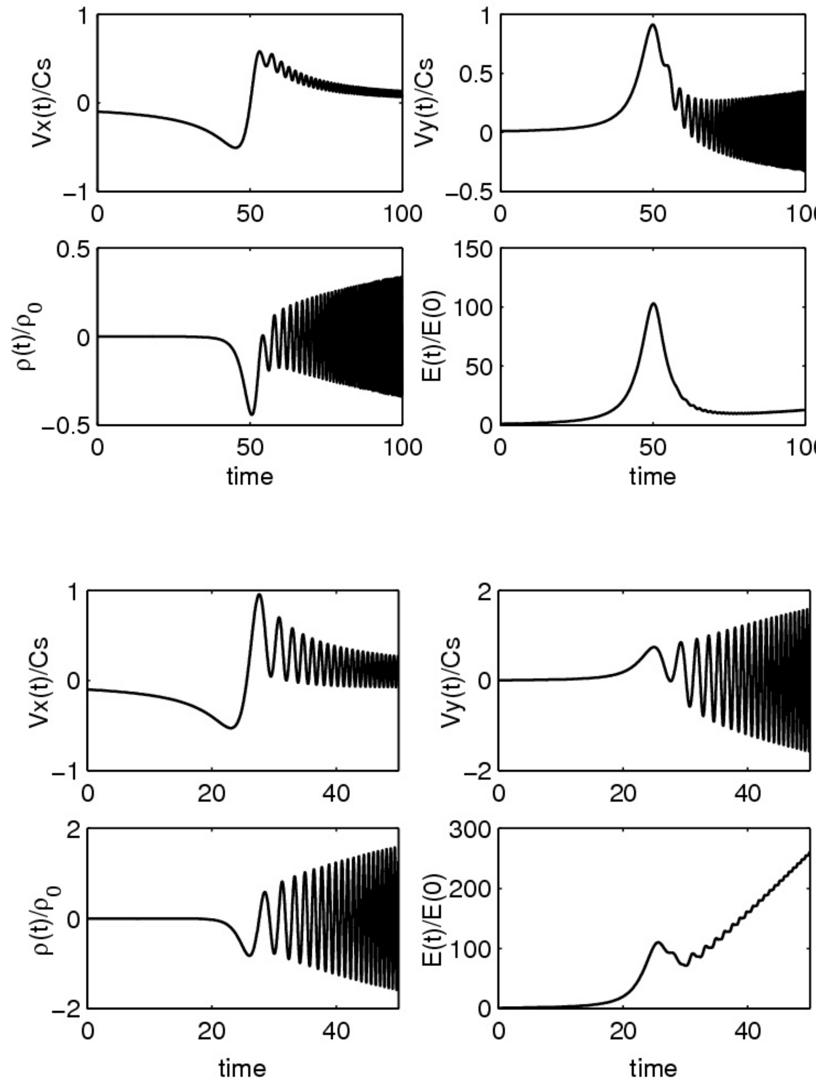
Evolution of vortex SFH in compressible shear flows:

Upper panels:

$A / c_s k_x = 0.2$

Lower panels:

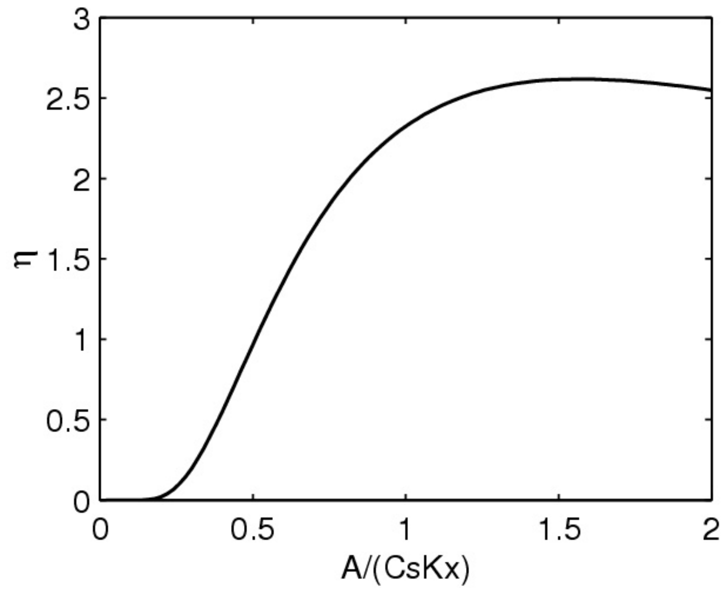
$A / c_s k_x = 0.4$



❸ Linear mode conversion

$$\eta \equiv \frac{E^{(\mathrm{w})}(t_+^*)}{E^{(\mathrm{v})}(t_-^*)}$$

Generated wave amplitude as the function of the velocity shear rate



Generated waves can have more energy then the source vortex: vortical perturbations only trigger the wave excitation while the energy is supported by the mean shear flow

❸ Linear mode conversion

DNS

Dynamics of the localized packet of vortex perturbations in shear flows

Initial values of perturbations corresponding to the acoustic free perturbations

Two different geometries of initial packets

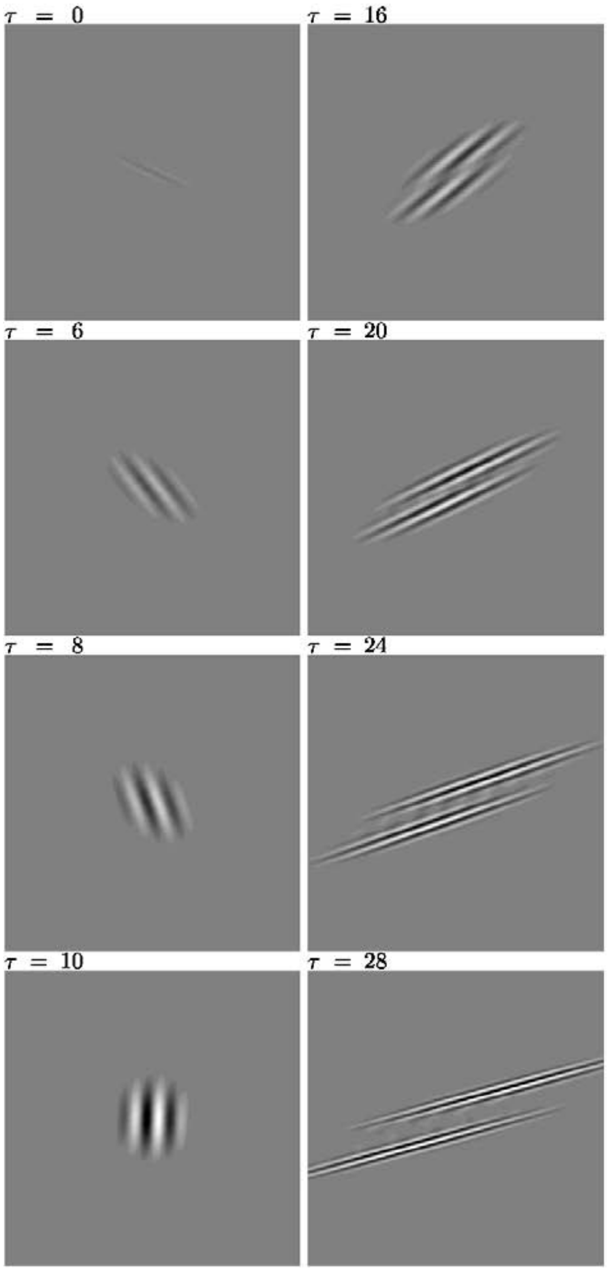
Initial amplitudes are small enough to ensure linear character of the process

❸ Linear mode conversion

Localized vortex packet with linear geometry

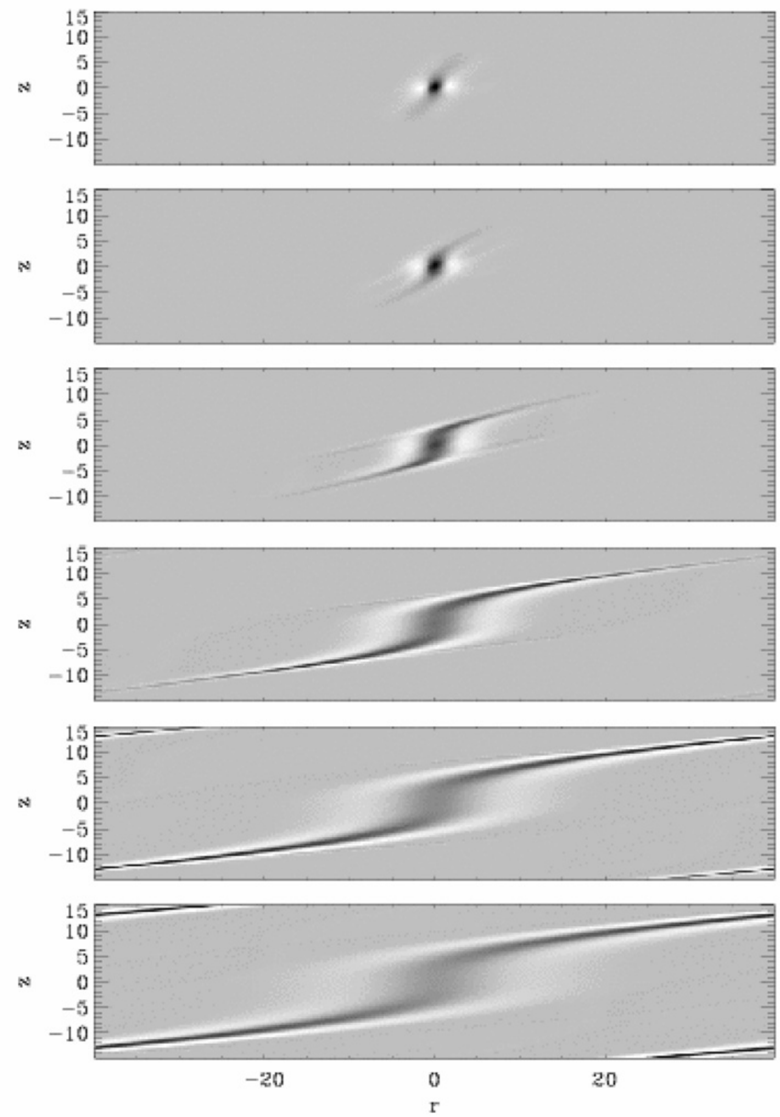
Enhencement of the vortex packet amplitude is followed by the wave excitation

Excited waves propagate in the opposite directions



❸ Linear mode conversion

Ring-type vortex



❸ Linear mode conversion

Non-resonant generation of acoustic waves from vortices

* Necessary condition for wave generation $k_y(0)/k_x > 0$

* Wave excitation occurs when $k_y(t) = 0$

* Waves are generated with zero density, cross-stream velocity and maximal streamwise velocity

* Excited waves are fed by the mean shear flow energy

Mode conversion contributes to the linear limit of the aerodynamic sound generation.

It tells the self consistent form of the aerodynamic variable and the source term, and shows the spatial correlation of the source flow and the excited oscillations (while only time correlations are considered in acoustic analogy)

4 Mode conversion in MHD shear flows

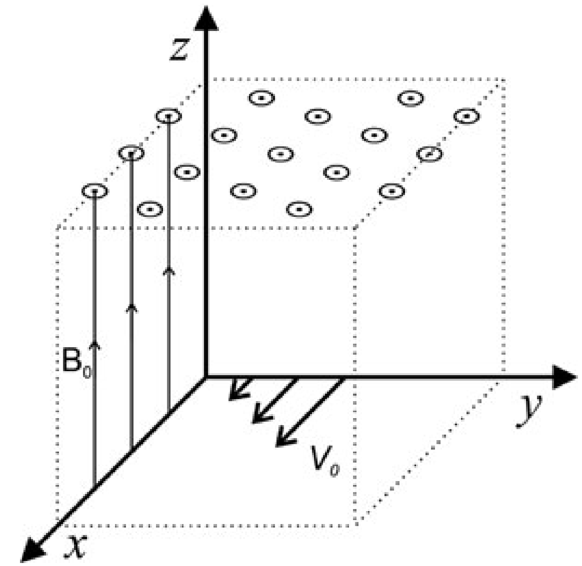
Model

Horizontal shear flow in vertical uniform magnetic field:

$\mathbf{V}_0 = (Ay,0,0)$

$\mathbf{B}_0 = (B_0,0,0)$

$d/dz\ \Phi = 0$



Compressible 3D ideal unbounded shear flow

4 Mode conversion in MHD shear flows

Coupling

Two aperiodic and a magnetosonic wave modes

$$\omega^2 = 0,$$

$$\omega_{\rm ms}^2 = (c_s^2 + V_A^2)k^2.$$

Vortex mode

$$I_1 = k_x v_y(t) - k_y(t) v_x(t) - A\frac{\mathrm{i}\rho(t)}{\rho_0},$$

Magneto-pressure mode

$$I_2 = \frac{\mathrm{i}\rho(t)}{\rho_0} - \frac{\mathrm{i}b_z(t)}{B_0}.$$

Wave sources:

$$\frac{\mathrm{d}^2 v_x(t)}{\mathrm{d}t^2} + (c_s^2 + V_A^2)k^2(t)v_x(t) = -(c_s^2 + V_A^2)k_y(t)I_1 - AV_A^2 k_y(t)I_2,$$

4 Mode conversion in MHD shear flows

Both aperiodic modes are able to excite ms. waves

$$\frac{E_{ms}^{(1)}(t^*)}{E_{ms}^{(2)}(t^*)} = \frac{(c_s^2+V_A^2)^2}{c_s^2 V_A^4 A^2}\frac{I_1^2}{I_2^2} = \frac{(1+\beta)^2}{\beta^2 R^2}\frac{I_1^2}{c_s^2 k_x^2 I_2^2},$$

Ratio of the effiencies of wave generation

Magneto-mechanical mode is a dominant wave generator when: $\beta >> 1, R > 1$

Vortex mode dominates when: $\beta < 1, R <1$

Convectively unstable shear flows

Model

Unbounded compressible 3D parallel shear flow in unofrom vertical gravity:

$$V_0 = (Ay, 0, 0)\ ,\ g = (0, 0, -g)\ ,\ g = const.$$

Vertical exponential stratification

$$\frac{P_0(z)}{P_0(0)} = \frac{\rho_0(z)}{\rho_0(0)} = \exp(-k_H z)$$

Convectively unstable flow

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Linear spectrum

1. Acoustic wave mode *(gravity modified)*
2. Convective mode *(exponential growth)*
3. Vortex mode *(algebraic growth)*

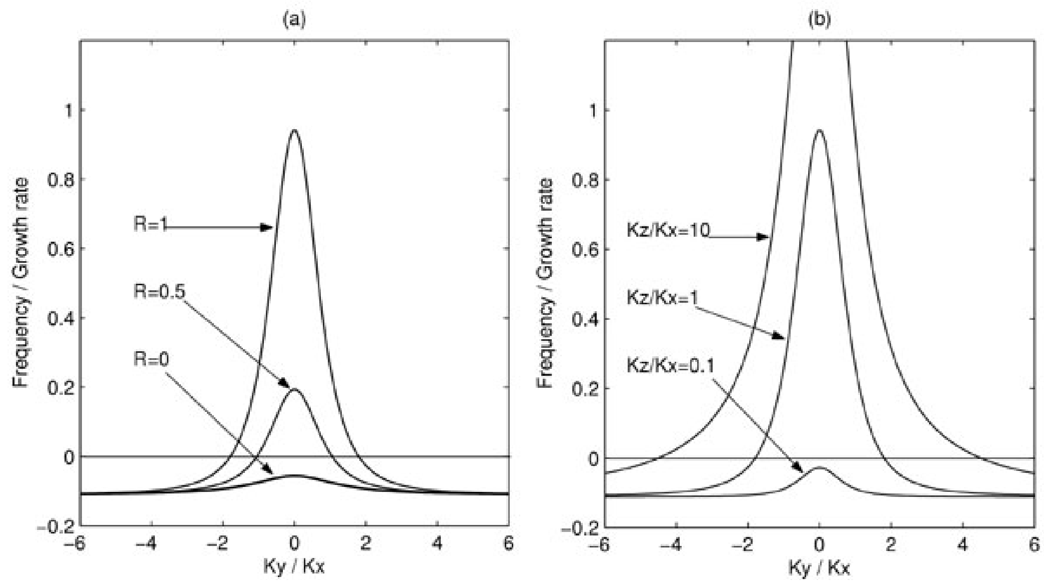
Small scale perturbations

$$k_z >> k_H$$

Constant vertical gravity, constant velocity shear

5 Convectively unstable shear flows

Neglecting the effect of acoustic waves and vortices on convective mode we study dynamics of unstable buoyancy perturbations separately.

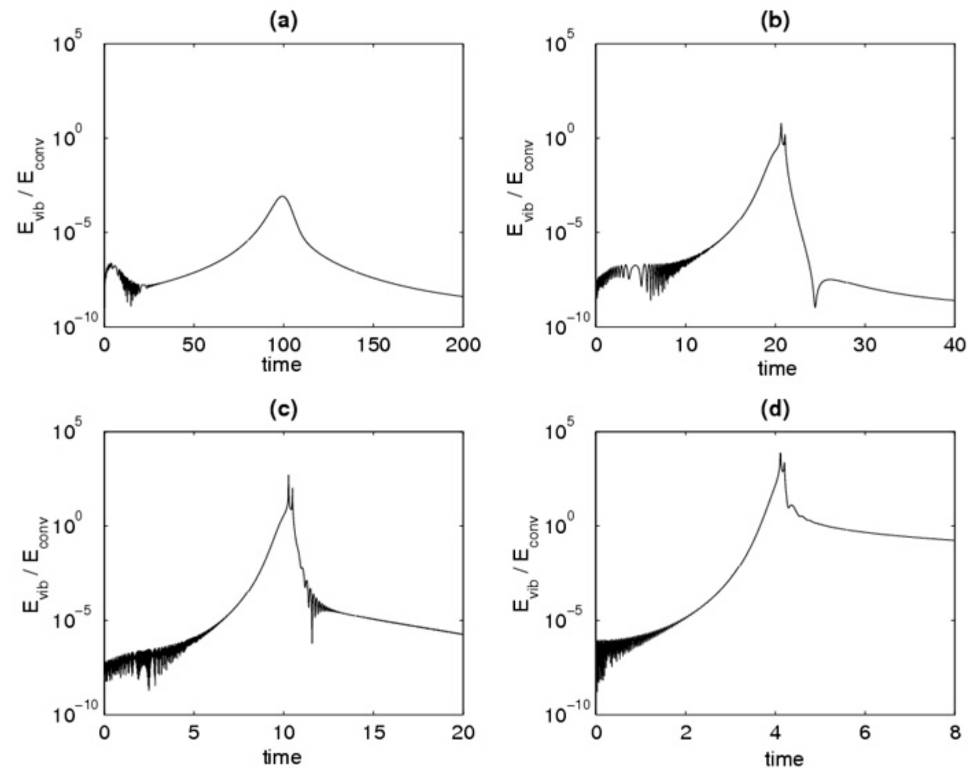


Velocity shear exerts a transient stabilizing effect on the spectral instability

❺ Convectively unstable shear flows

Mode coupling

Convectively unstable perturbations of buoyancy excite acoustic waves and the ratio of the vibrational to convective energies grows.



The ratio of the vibrational to the thermal energy of the SFH of perturbations (E_{vib}/E_{con}) vs time is shown at different velocity shear rates. Here $K_x = K_z = 10$, $K_y(0) = 200$ and $\sigma = -0.055$ ($\gamma = 0.95$). $R = 0.2, 1, 2, 5$ on the a, b, c and d graphs respectively. The time interval is chosen to show symmetric values of $K_y(\tau)$: $|K_y(0)| = |K_y(2\tau^*)|$ and $K_y(\tau^*) = 0$, where $\tau^* = 100, 20, 10, 4$ on the a, b, c and d graphs, respectively.

5 Convectively unstable shear flows

New channel of energy exchange between the g-modes and acoustic waves

Sound production in solar convection zone:

Mode conversion: buoyancy perturbations are able to excite acoustic waves with similar wave-numbers

Stochastic mechanism: generated frequencies are similar to the life-times of the source perturbations

Arguments for the shear flow wave production:

Stronger oscillations observed in intergranular dark lanes

Puzzling wave-number dependence of oscillations at fixed frequencies

❻ Differentially rotating flows

Model

Differentially rotating flow around the central gravitating object

$$V_0 = (0, r\Omega(r), 0),\ \mathbf{\Omega}(r) = (0,0,\Omega(r)),$$

$$c_s^2,\ H,\ P_0(r) = Const.$$

Vertical exponential stratification – constant gravity parameter

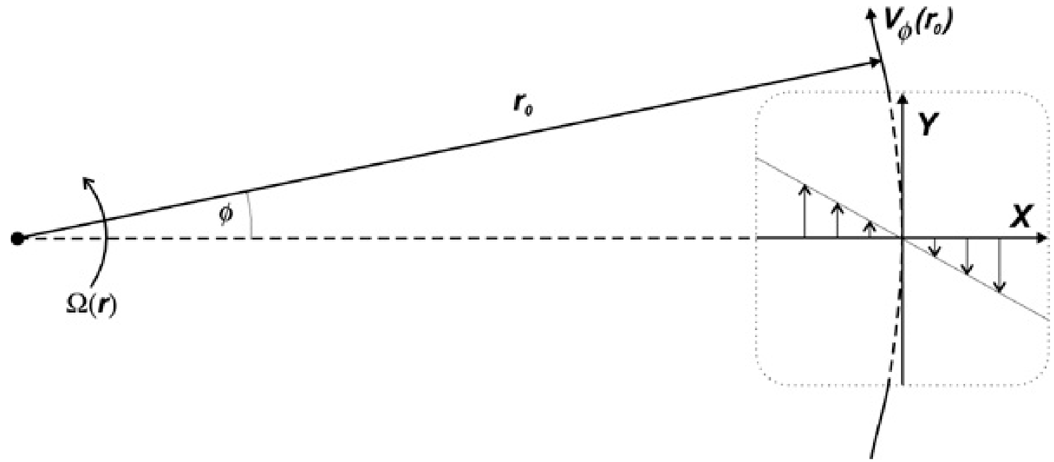
$$\frac{P_0(z)}{P_0(0)} = \frac{\rho_0(z)}{\rho_0(0)} = \exp\left(-\frac{|z|}{H}\right)$$

$$H = \frac{c_{\rm s}^2}{\gamma g}$$

Thin Keplerian disk model: $\Omega_{Kep}(r) \sim r^{-3/2}$

❻ Differentially rotating flows

Linear perturbations in local frame



1. High frequency acoustic waves
2. Low frequency density-spiral waves
3. Vortex mode

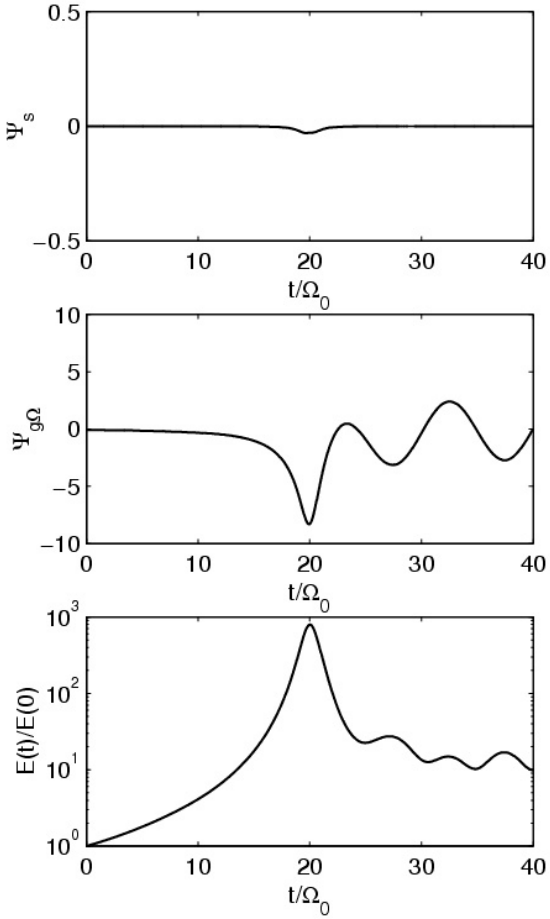
$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}t^2}+\omega_A^2\right\}\Phi_A(t)=0,$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}t^2}+\omega_f^2\right\}\Phi_f(t)=0,$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}t^2}+\omega_s^2\right\}\Phi_s(t)=0,$$

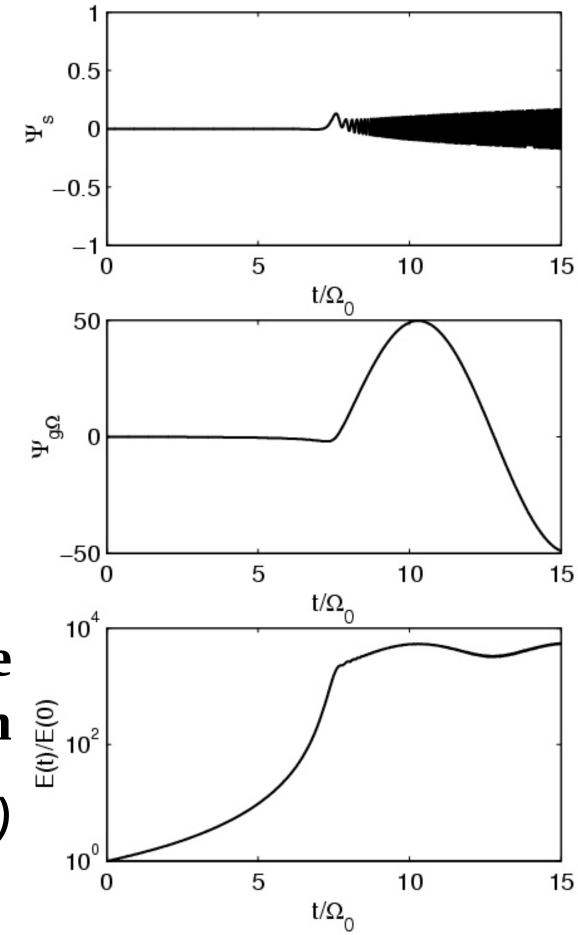
❻ Differentially rotating flows

Coupling



Excitation of density spiral waves

(A=3/4)



Double excitation

(A = 2)

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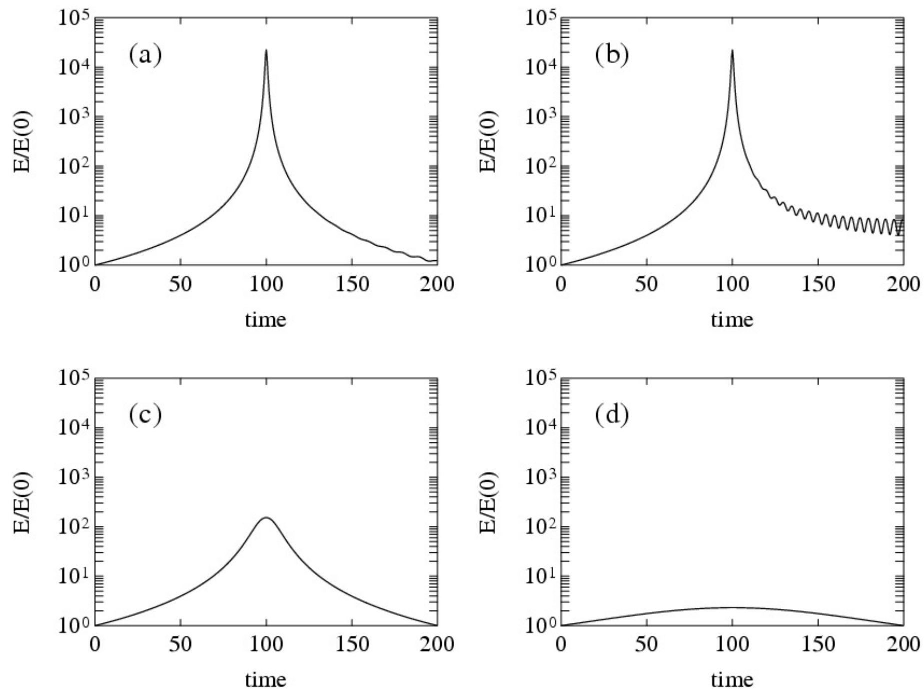
Transient amplification



Dynamics of the spectral energy densities of the perturbations in the Keplerian shear flows. Initial values of perturbations in all cases correspond to the purely vortex mode perturbations. Here $k_x(0)/k_H = -150$, $K_z = 10$ and $k_z/k_y = 0.01$ on graph a; $k_z/k_y = 0.1$ on graph b; $k_z/k_y = 10$ on graph c and $k_z/k_y = 100$ on graph d.

❻ Differentially rotating flows

Turbulence in HD accretion disks

Turbulence in spectrally stable flows:

subcritical transition (plane Couette flow)

Rotating flows:

Coriolis force introduces stabilizing effect

Turbulence in spectrally stable rotating flows?
Open issue

❻ Differentially rotating flows

Arguments toward the possibility of HD turbulence:

1. ***Analytic:***

   Renormalization of pressure perturbations reduces the dynamical system to the system that describes the plane shear flows

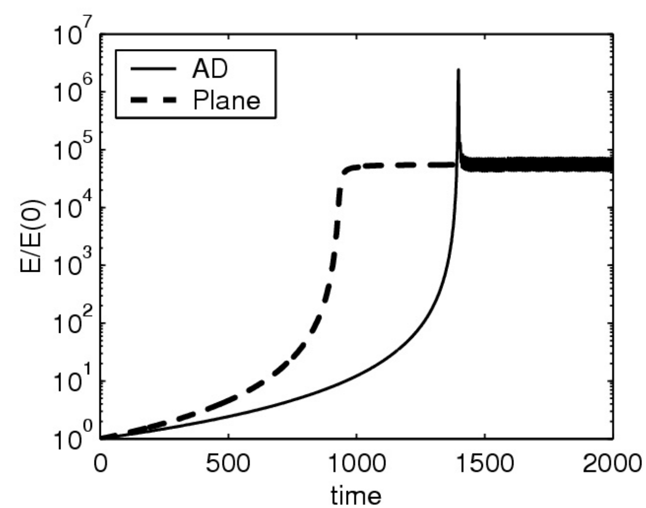
2. ***Numerical:***

   Growth rates match when increasing the Reynolds number.

Plane: $\mu = 2.25\ 10^3$

AD: $\mu = 4.41\ 10^6$

$$Re \sim \mu^{3/2}$$



❻ Differentially rotating flows

- Vertical gravity is necessary for the existence of vortex mode (Turbulence is 3D)
- Coriolis force increases the critical Reynolds number (in accretion disks Re ~ 10^{10})

Promising mechanism towards the turbulence in 3D compressible HD accretion disks:

a) Linear drift of wave-numbers

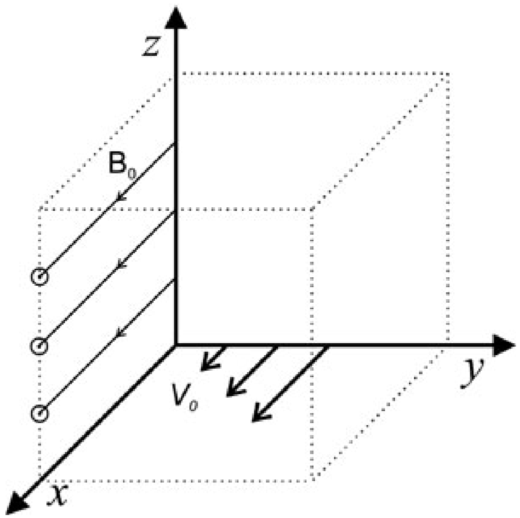
b) Transient growth

c) Viscous dissipation

d) Nonlinear processes that close the feedback loop

High Reynolds number 3D numerical calculations are required

Model



Horizontal shear flow in the uniform magnetic field along the streamlines:

$\mathbf{V_0}=(Ay,0,0)$, $\mathbf{B_0}=(B_0,0,0)$

Unbounded 3D ideal compressible MHD shear flow

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Linear spectrum

Dispersion equation in shearless limit:

$$\left(\omega^2 - V_A^2 k_x^2\right)\left(\omega^4 - (c_s^2 + V_A^2)k^2\omega^2 + V_A^2 c_s^2 k_x^2 k^2\right) = 0.$$

Standard MHD spectrum: Fast, Slow Magnetosonic and Alfven waves

Introducing eigenfunctions

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \omega_A^2\right\}\Phi_A(t) = 0,$$

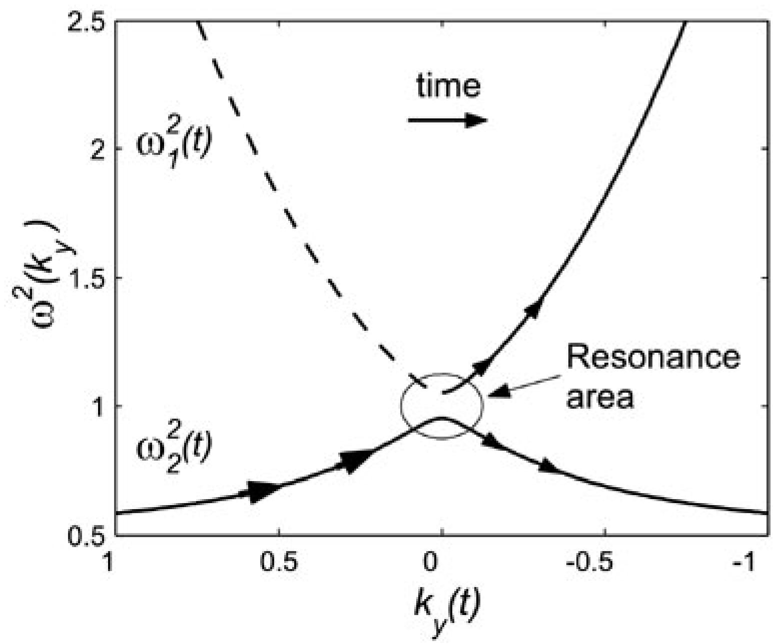
$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \omega_f^2\right\}\Phi_f(t) = 0,$$

$$\left\{\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \omega_s^2\right\}\Phi_s(t) = 0,$$

Coupling

Velocity shear induces coupling between the MHD modes.

Wave number variation: $k = k(t)$: $\omega = \omega(k) = \omega(t)$



MHD wave frequencies vary in time: waves can fall in resonance

Transformation of waves in MHD shear flows

Cold plasmas: $\beta = 0$

$$\frac{d^2\Phi_1(t)}{dt^2} + \omega_1^2(t)\Phi_1(t) = -\mathbf{\Lambda}\Phi_2(t),$$

$$\frac{d^2\Phi_2(t)}{dt^2} + \omega_2^2(t)\Phi_2(t) = \mathbf{\Lambda}\Phi_1(t),$$

Coupling of fast magnetosonic and Alfven waves

$\boldsymbol{\omega}$ – modified frequencies

Φ – normilized eigenfunctions

Λ – Coupling form

Mechanical Analogy

Wave resonance:

1. Existence of a degeneracy region

$$|\omega_1^2(t) - \omega_2^2(t)| < \left| \frac{\mathbf{\Lambda}\Phi_i(t)}{\Phi_i(t)} \right|$$

2. Slow pass condition

$$\left| \frac{\mathrm{d}\omega_i(t)}{\mathrm{d}t} \right| \ll \left| \frac{\mathbf{\Lambda}\Phi_i(t)}{\Phi_i(t)} \right|$$

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Regimes

☑ Fast magnetosonic and Alfven waves

$$\omega_f^2 \approx \omega_A^2 \qquad (\beta < 1,\ k_z / k_x \ll 1)$$

☑ Alfven and slow magnetosonic waves

$$\omega_A^2 \approx \omega_s^2 \qquad (\beta > 1)$$

☑ Fast, slow magnetosonic and Alfven waves

$$\omega_f^2 \approx \omega_A^2 \approx \omega_s^2 \qquad (\beta = 1,\ k_z / k_x \ll 1)$$

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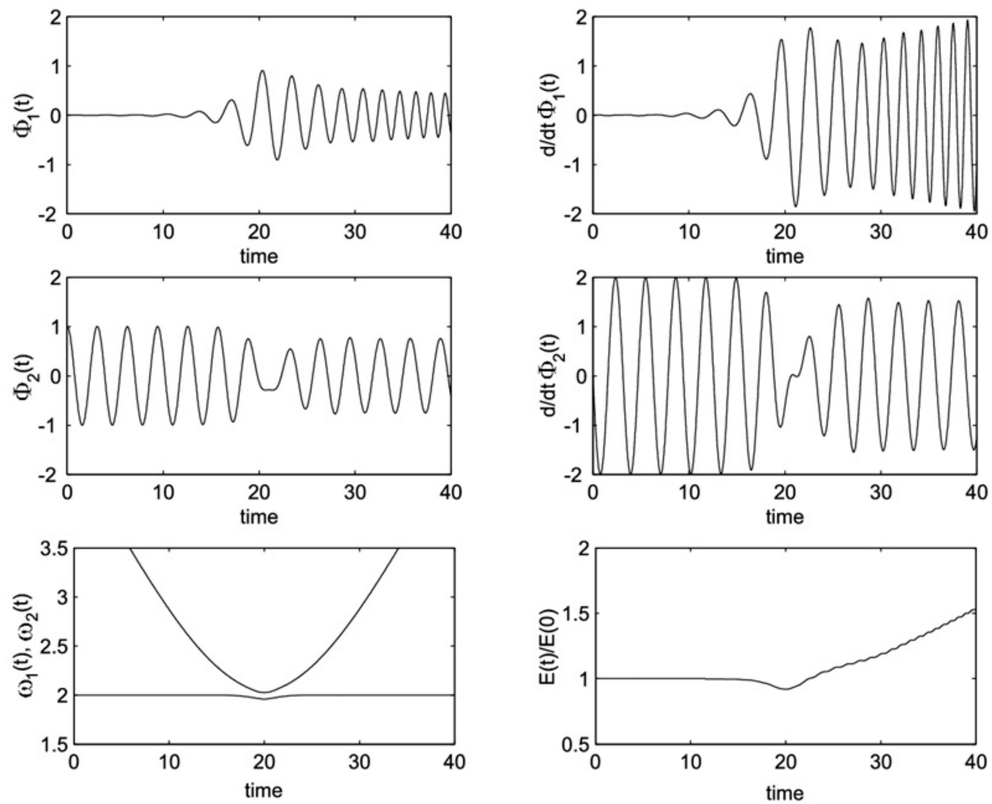
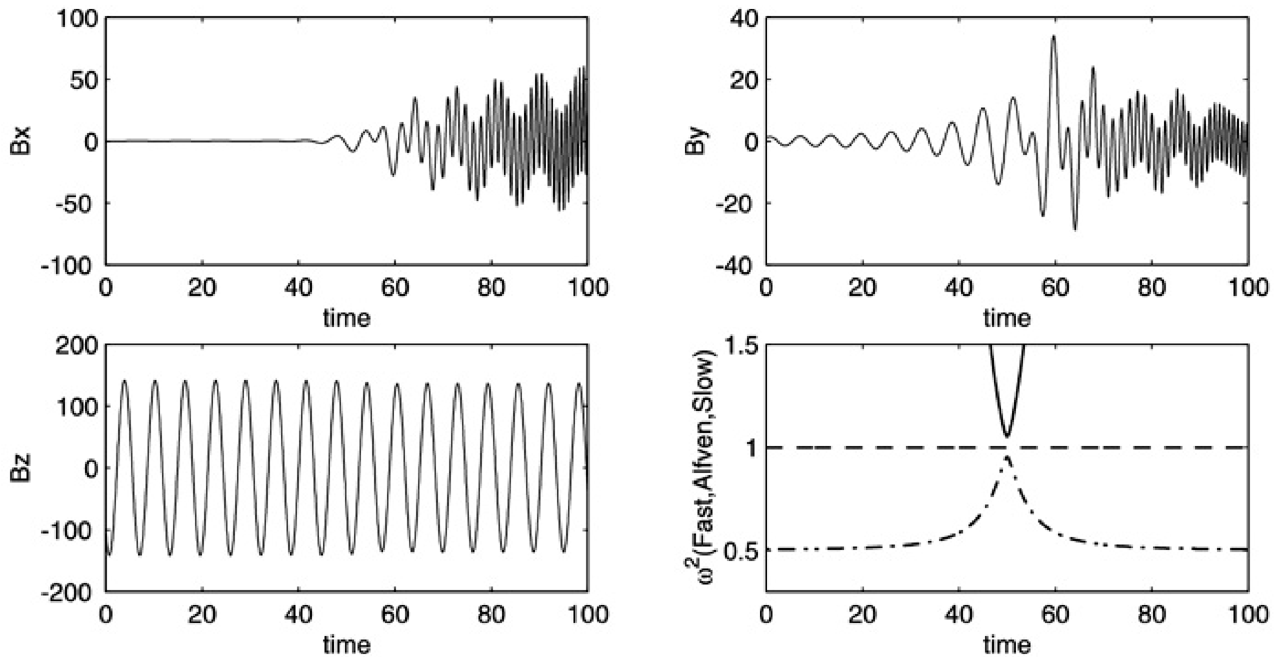
Numerical Results



Transformation of the Alfven into the fast magnetosonic wave

$k_y(0)/k_x = 2$, $k_z/k_x = 0.25$, $A/(V_A k_x) = 0.025$

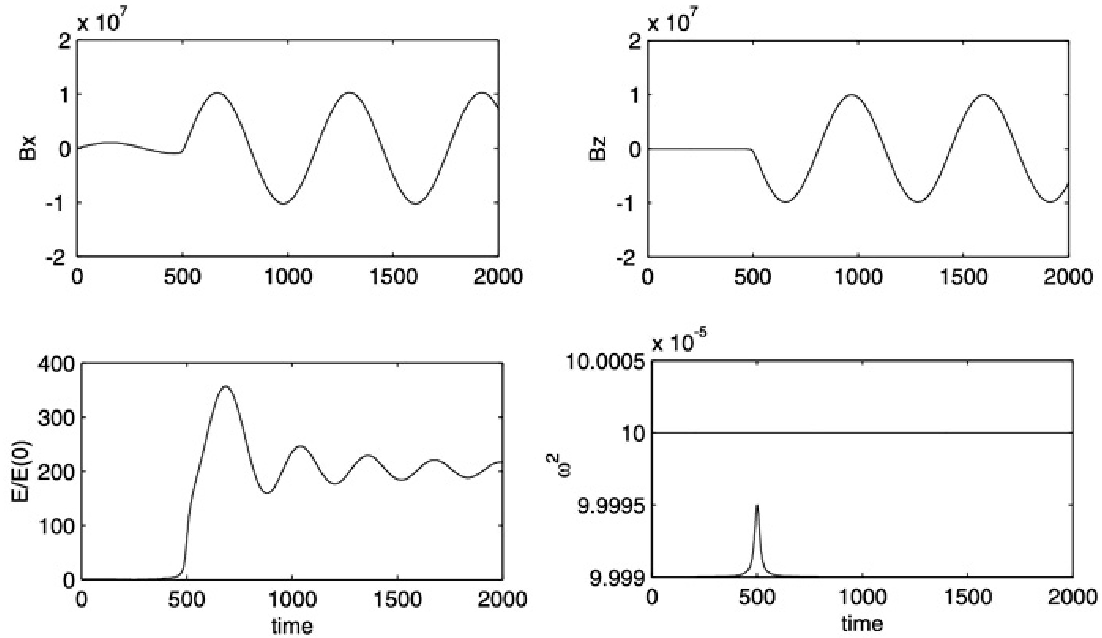
Double transformations



The case of double transformation: Alfvenic perturbations generates fast and slow magnetosonic waves simultaneously.

$\beta = 1, \quad k_y(0)/k_x = 5, \quad k_z/k_x = 0.05, \quad A/(C_s k_x) = 0.1$

7 Transformation of waves in MHD shear flows



Amplification and transformation of the slow magnetosonic waves:

Wave transformations + Transient amplification

Wave period exceeds the shearing time:

$$\frac{1}{\omega_s^2(t)} > \frac{1}{A^2}$$

Summary

Linear resonant interaction of MHD wave modes in shear flows: reciprocal wave transformations.

Intensity of the energy exchange between waves depends on how well the resonance conditions are realized.

High shear rates lead to the fast passage of the resonance area and rapid decrease in the wave transformation rates.

Well studied phenomenon in different astrophysical situations

Summary

Nonmodal analysis of flows with velocity inhomogeneities

Linear mode interactions

Non-resonant mode conversion

vortex - acoustic wave; vortex - ms wave;

g-mode – acoustic wave;

Resonant wave transformations

MHD wave transformations

Transient amplification

plane shear flows, slow ms wave;

differentially rotating flows;

8 Summary

Aerodynamic sound generation:

constraints in the linear limit
verified results of nonmodal analysis by DNS

Acoustic wave generation in solar convection zone

Interplay of modal and nonmodal processes
properties of waves generated by velocity inhomogeneities

Accretion disk theory:

dynamics of vortices, excitation of density-spiral waves, root to the transition to HD turbulence

Questions?