Natural and Engineering Nonuniform Flows as Dynamical Systems: Issues and Challenges

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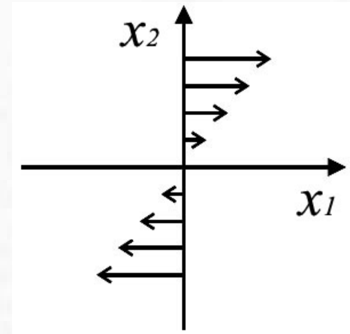
Outline

- **Nonuniform flows: shear flow non-normality**
- **Aerodynamic sound generation**
- **Dynamics of astrophysical flows**

  - *Stability of Accretion disks*
  - *Dynamics of Protoplanetary flows*
- **Shear flow turbulence**

  - *Bypass transition: nonlinear transverse cascade*
  - *Turbulence control*
- **Summary**

Nonuniform Hydrodynamic Flow



Nonuniform flow with constant velocity shear: $\bar{U}_i(\mathbf{x}) = (Ax_2, 0, 0)$

Dynamics of linear compressible perturbations:

$$\left(\frac{\partial}{\partial t} + Ax_2\frac{\partial}{\partial x_1}\right)U_i' = -c_s^2\frac{\partial}{\partial x_i}\frac{\rho'}{\rho_0} + \nu\frac{\partial^2}{\partial x_k \partial x_k}U_i'$$

$$\left(\frac{\partial}{\partial t} + Ax_2\frac{\partial}{\partial x_1}\right)\frac{\rho'}{\rho_0} = -\frac{\partial U_k'}{\partial x_k}$$

Eigenfunction/Eigenvalue analysis:

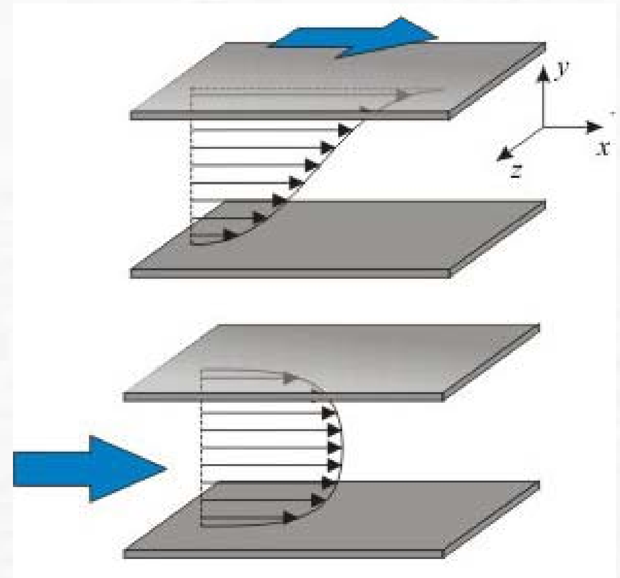
$u_j(\mathbf{x}, t) = \varphi_j(\mathbf{x}, \omega)e^{\mathrm{i}\omega_j t}$ Linear modes (stability)

Shear Flow Stability

Reynolds number: $\mathrm{Re} = \mathrm{UL}/\nu$

Problem:

	Theory Re_{cr}	Experiment Re_{cr}
Couette Flow	∞	350
Poiseuille flow	5772	1000



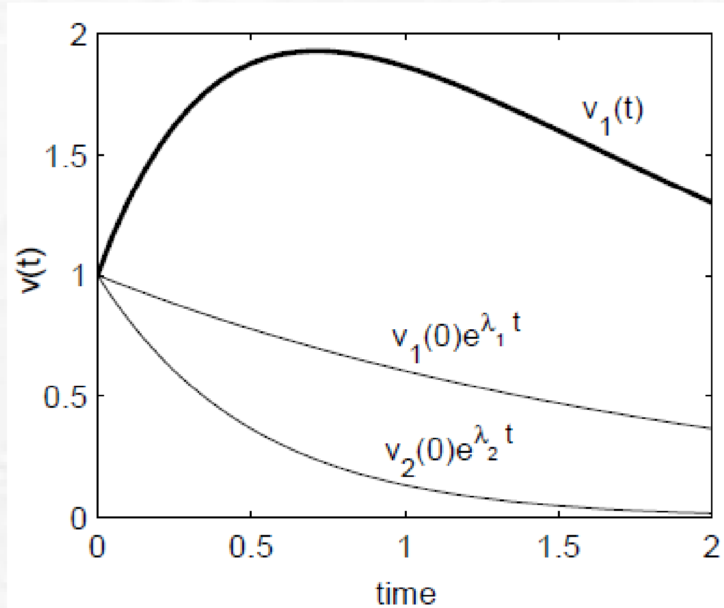
Solution:

Shear flow non-normality (non-Hermitian system)

Eigenfunction interference

Threfethen et al. Science, 1993

Reddy et al. SIAM J. Appl. Math. 1993



Algebraic growth, “Pseudospectrum”

Kelvin Mode Analysis

"Shearing sheet" transformation

$$x_1' = x_1 - Ax_2t \ , \ x_2' = x_2 \ , \ x_3' = x_3 \ , \ t' = t$$

"Kelvin modes"

$$\Psi_j'(\mathbf{x}, t) \propto \psi_j(\mathbf{k}(\mathbf{t}), t) \exp\left(\mathrm{i}\mathbf{k}(t)\mathbf{x}\right)$$

$$k_2(t) = k_2(0) - Ak_1t$$

Eigenmode analysis: Boundary value problem
Kelvin mode analysis: Initial value problem

Time evolution of spatial Fourier harmonics

$$\omega_j = \omega_j(\mathbf{k}(t)) = \omega_j(t)$$

Example: Linear MHD waves

$$\left\{\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right\}\rho + \rho(\nabla\cdot\mathbf{V}) = 0$$

$$\rho\left\{\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right\}\mathbf{V} = -\nabla P + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B}$$

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{V}\times\mathbf{B})$$

$$\nabla\cdot\mathbf{B} = 0$$

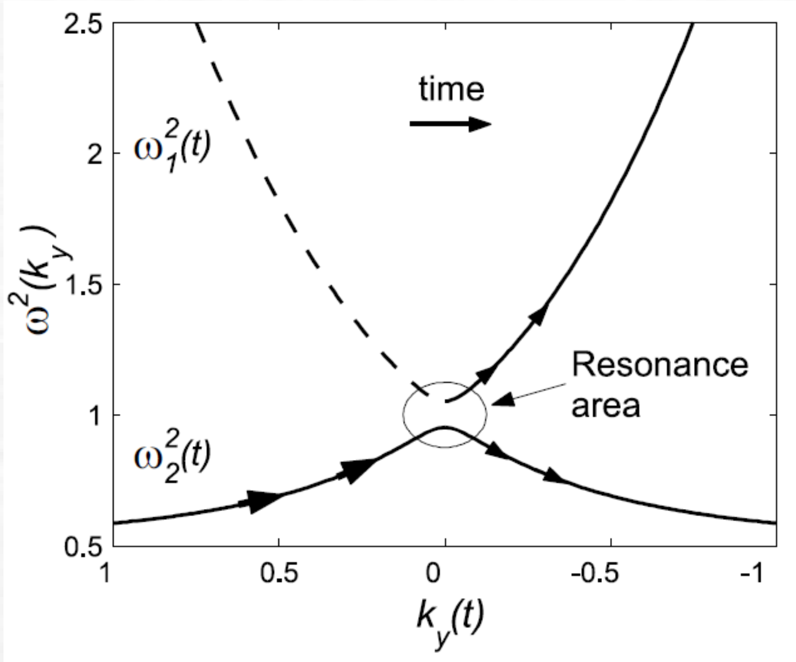
$$\left\{\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right\}P + \gamma P(\nabla\cdot\mathbf{V}) = 0$$

Magneto
Hydro
Dynamics

Linear coupling of modes:

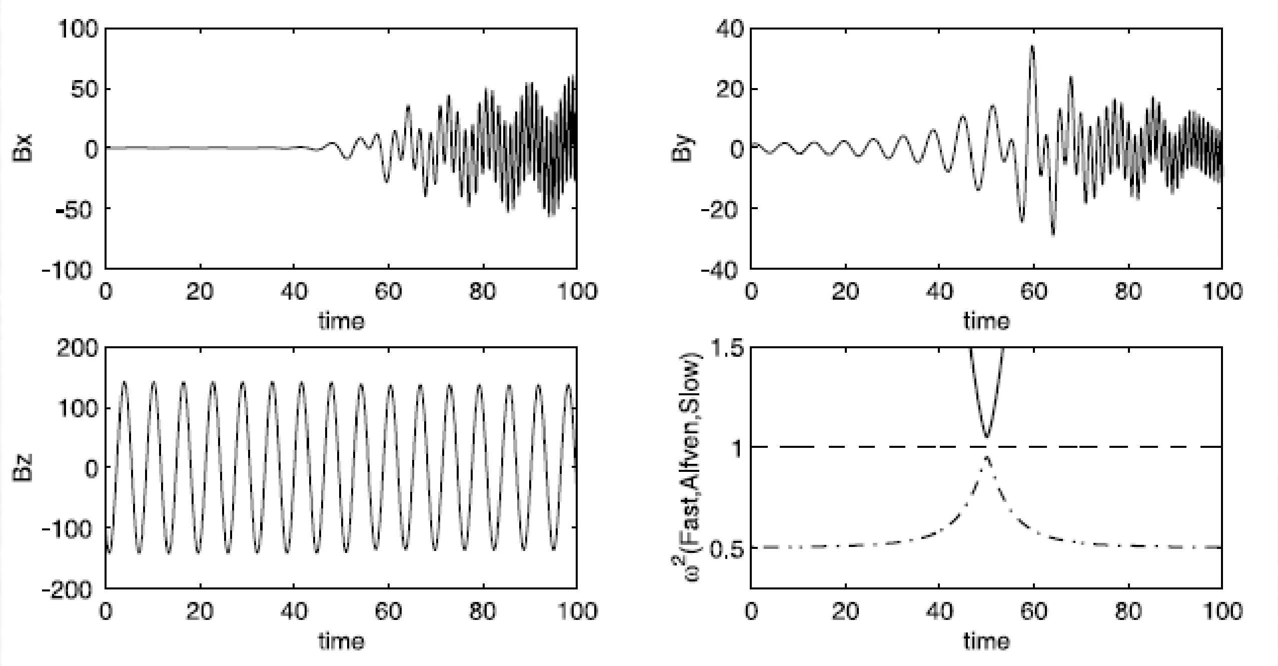
$$\frac{\mathrm{d}^2\Phi_1(t)}{\mathrm{d}t^2} + \omega_1^2(t)\Phi_1(t) = -\mathbf{\Lambda}\Phi_2(t),$$

$$\frac{\mathrm{d}^2\Phi_2(t)}{\mathrm{d}t^2} + \omega_2^2(t)\Phi_2(t) = \mathbf{\Lambda}\Phi_1(t),$$



High order systems: multiple modes

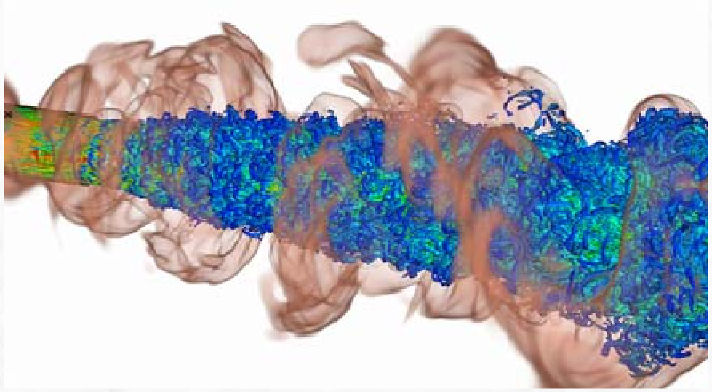
Triple resonance: Fast, Slow and Alfven waves;



More complex configurations:

- Effects of gravity (magnetoconvection);
- MHD anisotropy ($P_{||}, P_{\perp}$, mirror & fire-hose);

Aerodynamic Sound



Acoustic wave excitation

Wind & Jet noise, Solar & stellar oscillations.

Uniform flow U(**x**,t)=const.

Lighthill (1951):

$$\underbrace{\frac{\partial^2 \varrho'}{\partial t^2} - c_s^2 \frac{\partial^2 \varrho'}{\partial x_i \partial x_i}}_{\text{sound propagation}} = \underbrace{\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}}_{'\text{source}'}$$

Reynolds stress tensor: $T_{ij} = \varrho U_i U_j + P_{ij} - \delta_{ij} c_s^2 \varrho$

Acoustic Analogy:

“<u>Wave Equation</u>” = “<u>Nonlinear Source</u>”

Aerodynamic Sound in Shear Flows

Constant shear flow: Ux = Ay.

Linear wave modes

Acoustic Wave:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} v_x^{(\mathrm{w})}(t) + c_s^2 k^2 v_x^{(\mathrm{w})}(t) = 0$$

Vortex:

$$k^2 v_x^{(\mathrm{v})} = k_y I.$$

Shear flow:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} v_x(t) + c_s^2 k^2(t) v_x(t) = c_s^2 k_y(t) I,$$

Non-resonant vortex-wave mode conversion

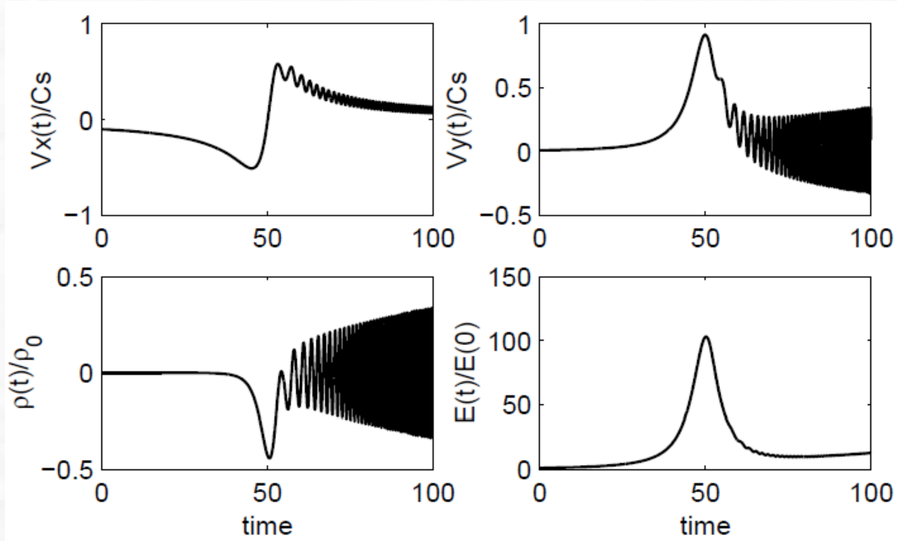
Chagelishvili et al. PRL 1997

Aerodynamic Sound in Shear Flows

Moderate shear: *$A/c_s k_x = 0.4$*

Asymmetric coupling:

- Vortex → Wave
- Vortex (PV invariant);



Acoustic excitation in shear flows:

$$\mathcal{L}\Psi = \mathcal{S}(\Psi).$$

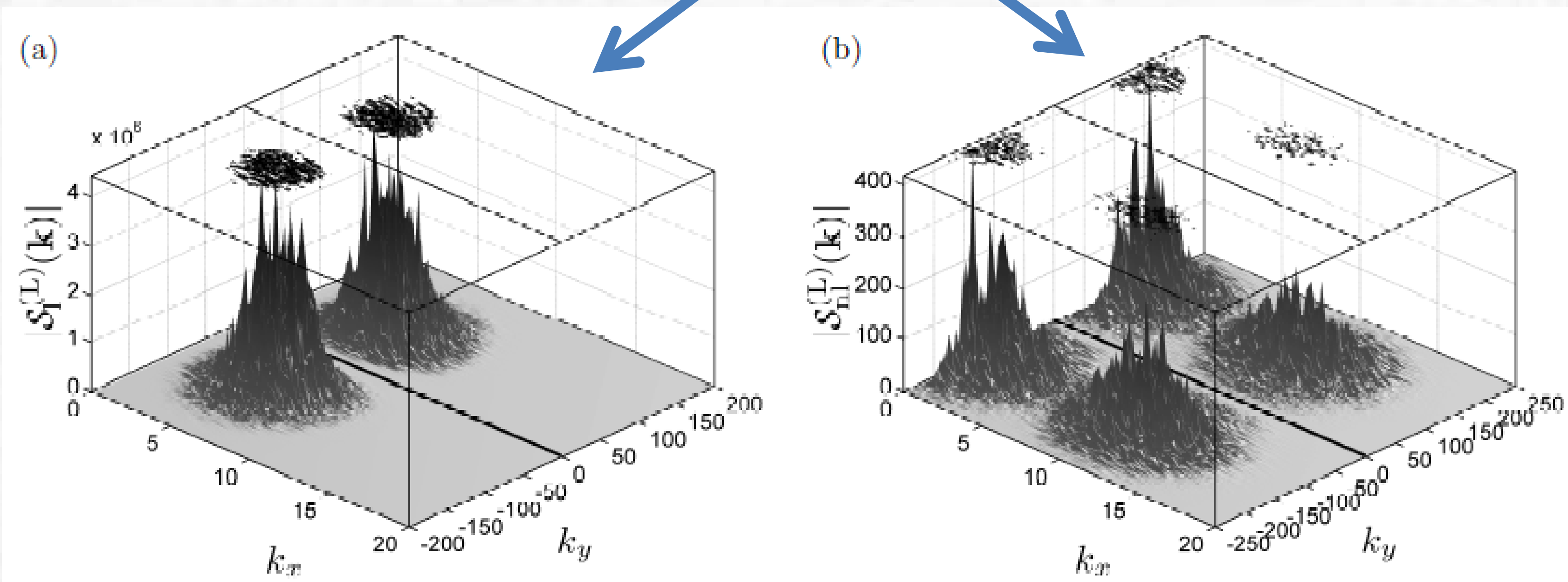
$$\mathcal{S}_{\rm l}^{(\rm L)} = \frac{\partial^2}{\partial x^2}\left[\rho' A^2 y^2 + 2\rho_0 v_x A y\right] + 2\frac{\partial^2}{\partial x \partial y}\left[\rho_0 v_y A y\right]$$

$$\mathcal{S}_{\rm nl}^{(\rm L)} = \frac{\partial^2}{\partial x^2}\left[\left(\rho_0 + \rho'\right) v_x'^2 + 2\rho_0 v_x A y\right] + 2\frac{\partial^2}{\partial x \partial y}\left[\left(\rho_0 + \rho'\right) v_x v_y + \rho' v_y A y\right] + \frac{\partial^2}{\partial y^2}\left[\left(\rho_0 + \rho'\right) v_y^2\right]$$

Aerodynamic Sound in Shear Flows

Direct Numerical Simulations (`PLUTO code`)

"Linear Source" $\mathcal{S} = \mathcal{S}_{\rm l} + \mathcal{S}_{\rm nl}$ "Nonlinear Source"



- Intrinsically different type of sources;
- S_l - wave-numbers, S_{nl} - frequencies;
- Vital to estimate far field acoustic field (noise);

Astrophysical Disks

Flows around central gravitating object:

Keplerian flow

$$\Omega^2(\mathbf{r}) = \frac{GM}{(r^2 + z^2)^{3/2}},$$

$$\Phi(\mathbf{r}) = -\frac{GM}{\sqrt{r^2 + z^2}}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho V_r) + \frac{1}{r}\frac{\partial}{\partial \phi}(\rho V_\phi) + \frac{\partial}{\partial z}(\rho V_z) = 0$$

$$\frac{\partial V_r}{\partial t} + (\mathbf{V} \cdot \nabla) V_r - \frac{V_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r}$$

$$\frac{\partial V_\phi}{\partial t} + (\mathbf{V} \cdot \nabla) V_\phi + \frac{V_r V_\phi}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \phi}$$

$$\frac{\partial V_z}{\partial t} + (\mathbf{V} \cdot \nabla) V_z = -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial \Phi}{\partial z}$$

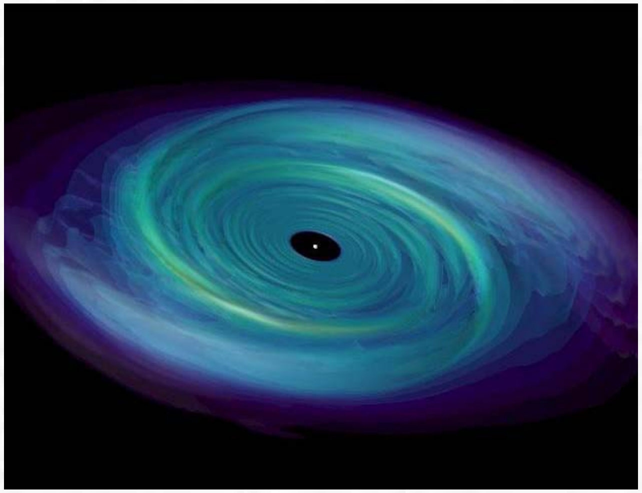
Differentially rotating axially symmetric flow

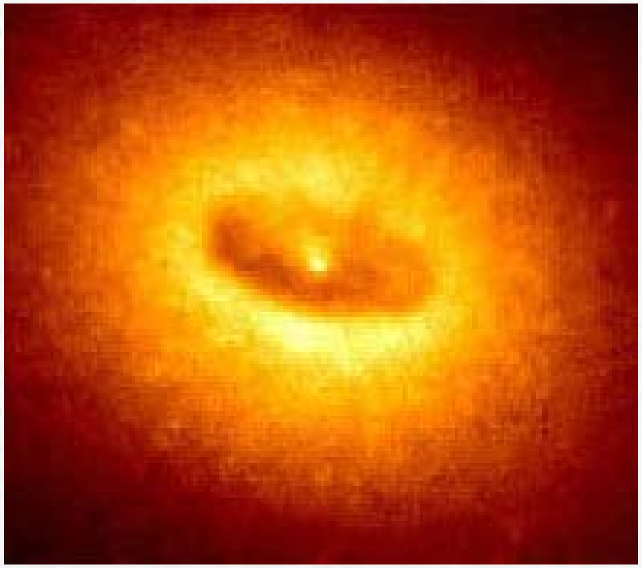
- Curvature terms;
- Radial shear terms;

Accretion Disk Turbulence

Accretion disks around compact objects

Most luminous objects in the Universe





Luminosity

Gravitational energy

Accretion

Angular momentum transport

Anomalous viscosity (α-model)

Turbulence

Linear theory: $Re_{cr}=\infty$

Observations: $Re >> 10^{10}$

Turbulence transition?

Dynamics of Protoplanetary Flows

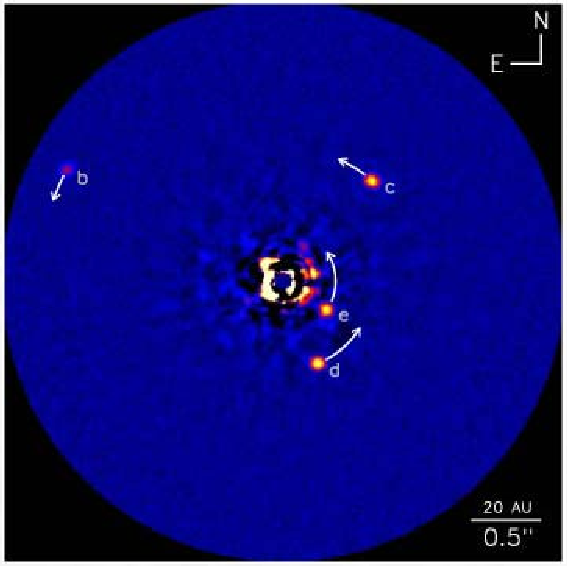
Rotating flows around young stars:

Planet formation areas

Exoplanets found: 1933 (20.07.15)

Problem:

Fast formation of planetesimals



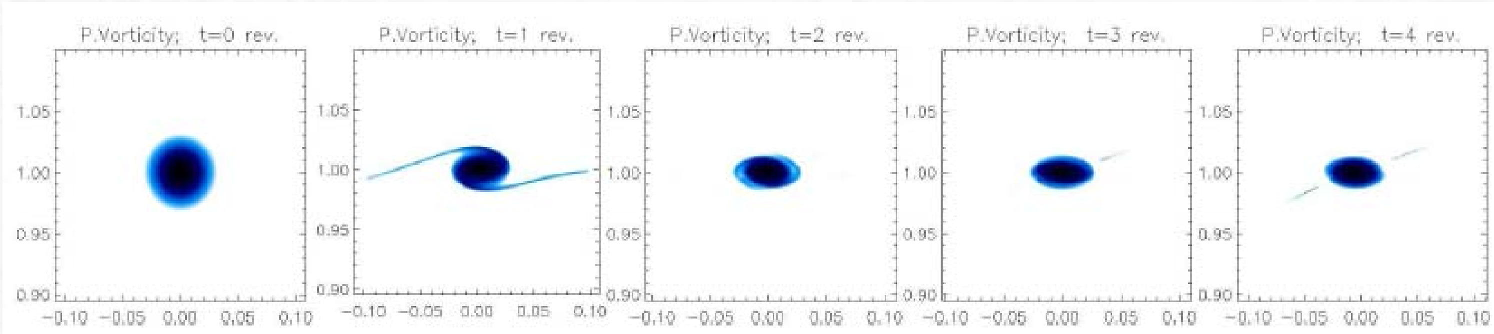
Important Aspects:

- Spiral-Density waves;
- Anticyclonic vortices;
- Spiral shock waves;

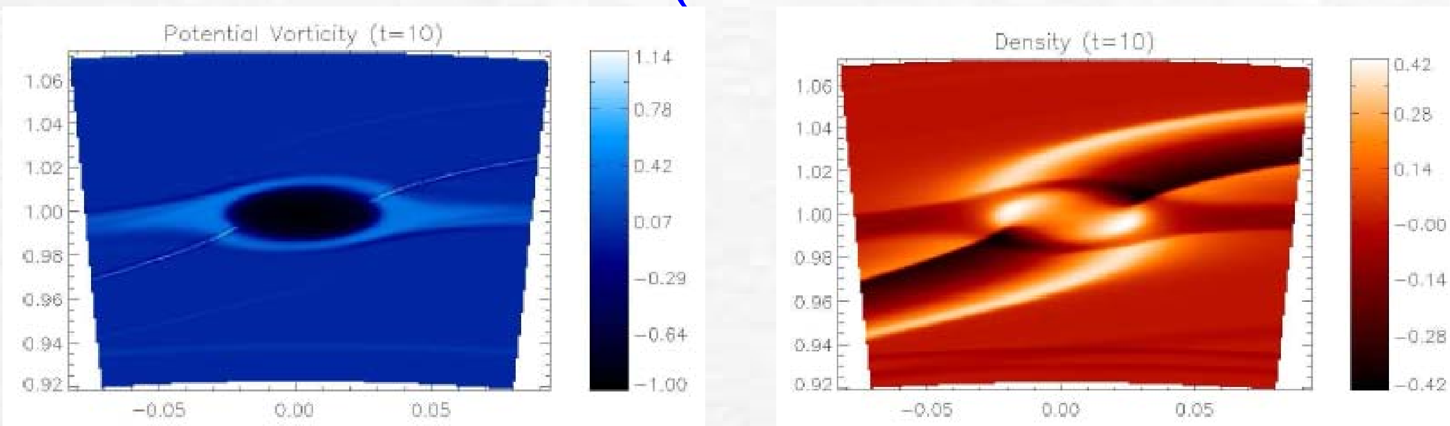
Dynamics of Protoplanetary Flows

Direct Numerical Simulations (Riemann/Godunov)

- Nonlinear adjustment of anticyclonic vortices;



- Self-sustained vortex (nonlinear attractor)



Stable vortex solution in Keplerian flow?

Incompressible Turbulence

Incompressible Navier-Stoke equation

$$\begin{aligned}\frac{\partial V_i}{\partial t} + V_k \frac{\partial}{\partial x_k} V_i &= -\frac{1}{\rho_0}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 V_i}{\partial x_k \partial x_k} \\ \frac{\partial V_k}{\partial x_k} &= 0\end{aligned}$$

Spectral energy in shear flows:

$$\frac{\partial E_{\boldsymbol{k}}}{\partial \tau} - \hat{A}\frac{2k_x k_y}{k^2} E_{\boldsymbol{k}} + \frac{\partial}{\partial k_x}(-\hat{A} k_y E_{\boldsymbol{k}}) + R_e^{-1} k^2 E_{\boldsymbol{k}} = \hat{N}_{\boldsymbol{k}}$$

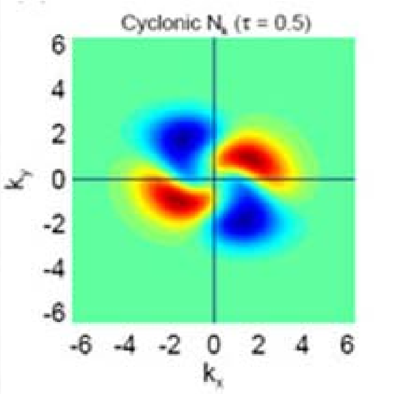
Nonlinear terms: $\Sigma_{\boldsymbol{k}} N_{\boldsymbol{k}} = 0$

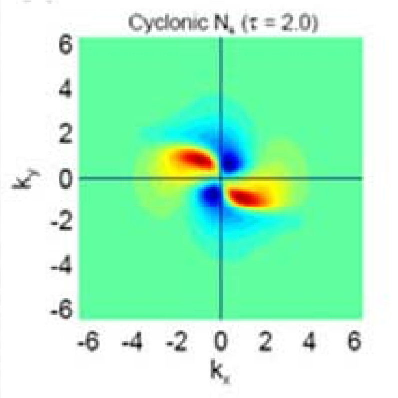
$$\hat{N}_{\boldsymbol{k}} = \sum_{\boldsymbol{k}=\boldsymbol{k}'+\boldsymbol{k}''} (k'_x k''_y - k''_x k'_y) k'^2 (\Psi^*_{\boldsymbol{k}} \Psi_{\boldsymbol{k}'} \Psi_{\boldsymbol{k}''} + \Psi_{\boldsymbol{k}} \Psi^*_{\boldsymbol{k}'} \Psi^*_{\boldsymbol{k}''})$$

Shear Flow Turbulence

Spectral description:

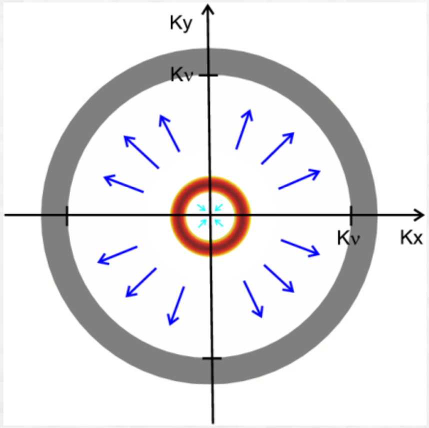
- Linear growth;
- Nonlinear redistribution;

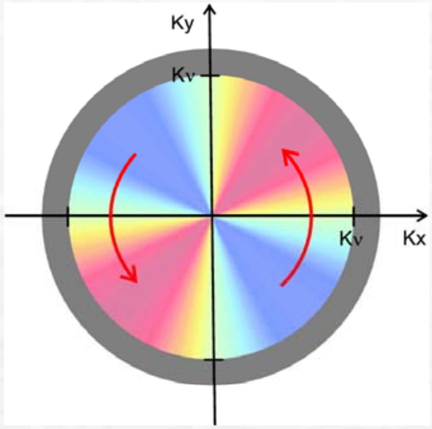




Numerical simulations: nonlinearities

Kolmogorov	Shear flows
"direct cascade"	"transverse cascade"





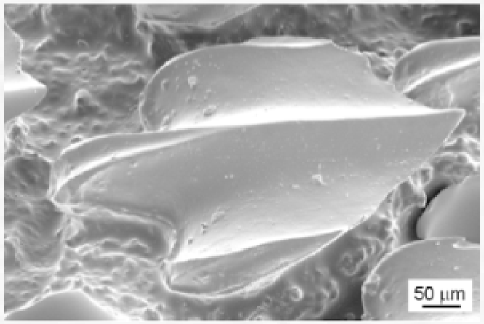
Turbulence Control

Fluid turbulence described by NS equation including stochastic force:

$$F_i(\mathbf{r},t) = \frac{\partial s_{ij}(\mathbf{r},t)}{\partial x_j}$$

Task: reduce turbulent fluctuations.

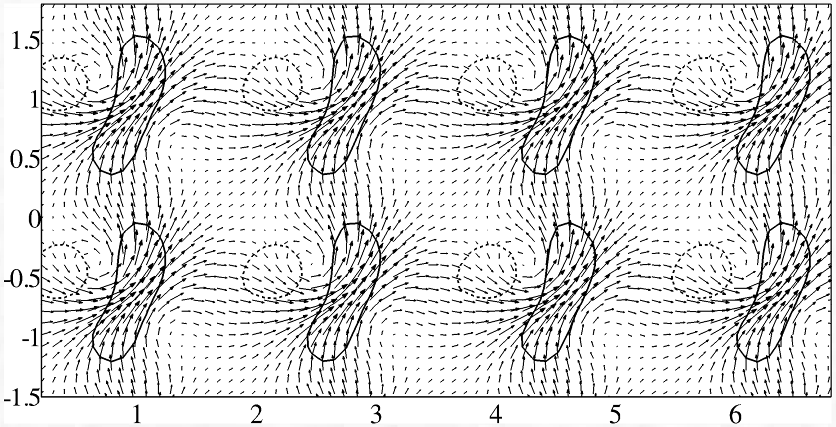
Examples in nature: *shark skin*



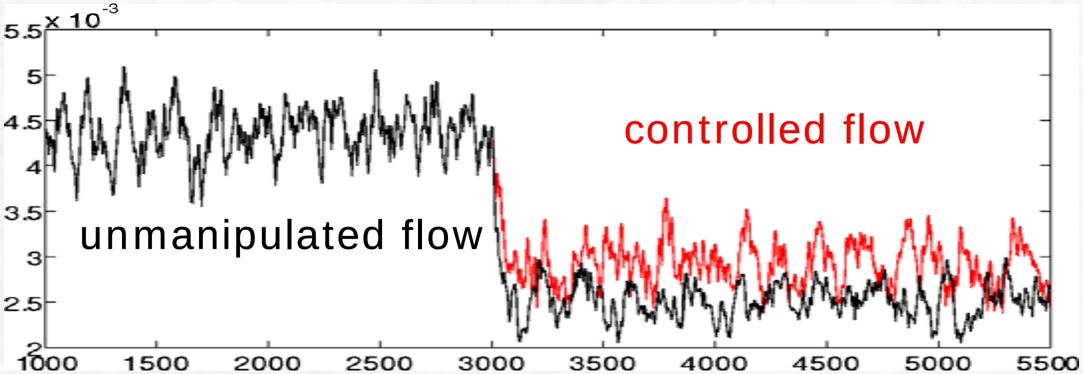
Method: Near wall forcing - create perturbations that rearrange flow and decrease turbulence power.

Turbulence Control

Seed velocity field created by forcing



Turbulence power vs time



The new drag reduction scheme:

generation of spanwise subflow to streamwise shear.

Khujadze, Oberlack, Chagelishvili PRL 2006

Summary

Examples of shear flows widely occurring in the nature and engineering applications;

Open Issues:

Aerodynamic sound:

Sound sources are tiny, but their result is crucial in far field; Linear analytic theory is crucial even for numerical simulation;

Accretion Disks:

Flows are know to be turbulent. Turbulence transition mechanism at high Reynolds numbers is still unclear.

Summary

Protoplanetary Disks:

Self-sustained anticyclonic vortices at nonlinear stages: No analytic solution;

Shear Flow Turbulence:

Results for incompressible turbulence. Effects of Compressibility needs to be addressed.

Turbulence Control:

Possibility of turbulent drag reduction is shown. A more efficient (realizable) control mechanisms needed to be explored.