ლექცია 8

N-Body Simulations

Primitive Approach:
Particle-Particle interactions **(PP)**

O(N^2) computation process

Methods:

- ✓ Mesh Based
- ✓ Tree Code
- ✓ Multipole Expansion

N-Body Simulations

Approaches:

- ☑ Particle-Mesh **(PM)**
- ☑ Particle-Particle, Particle-Mesh **(P3M)**
- ☑ Particle Multiple Mesh **(PM2)**
- ☑ Nested Grid Particle – Mesh **(NGPM)**
- ☑ Tree Code **(TC)**

Particle-Mesh **(PM)**

Approach: { Particles + Field }

1. Particle → Mesh

   Calculate Density Function

2. Mesh

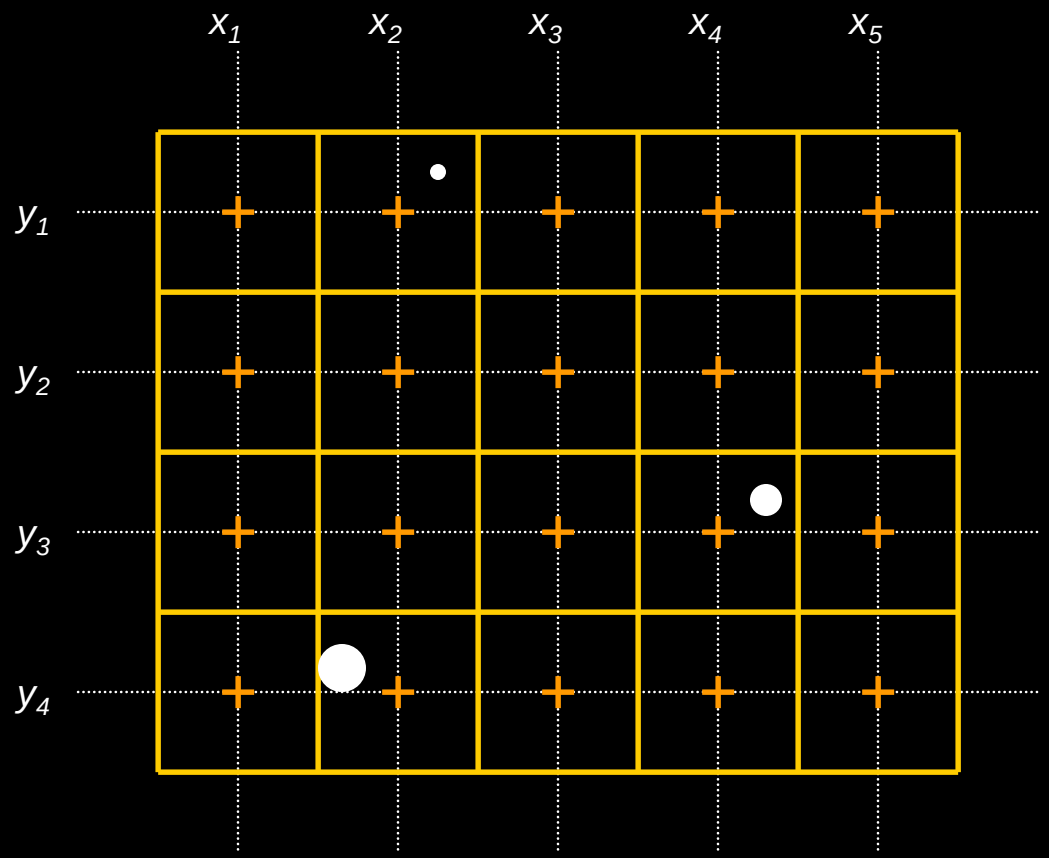
   Calculate Potential

3. Mesh → Particle

   Calculate Forces

Particle-Mesh **(PM)**

(N) Particles + (m^2) Mesh



Particle-Mesh **(PM)**

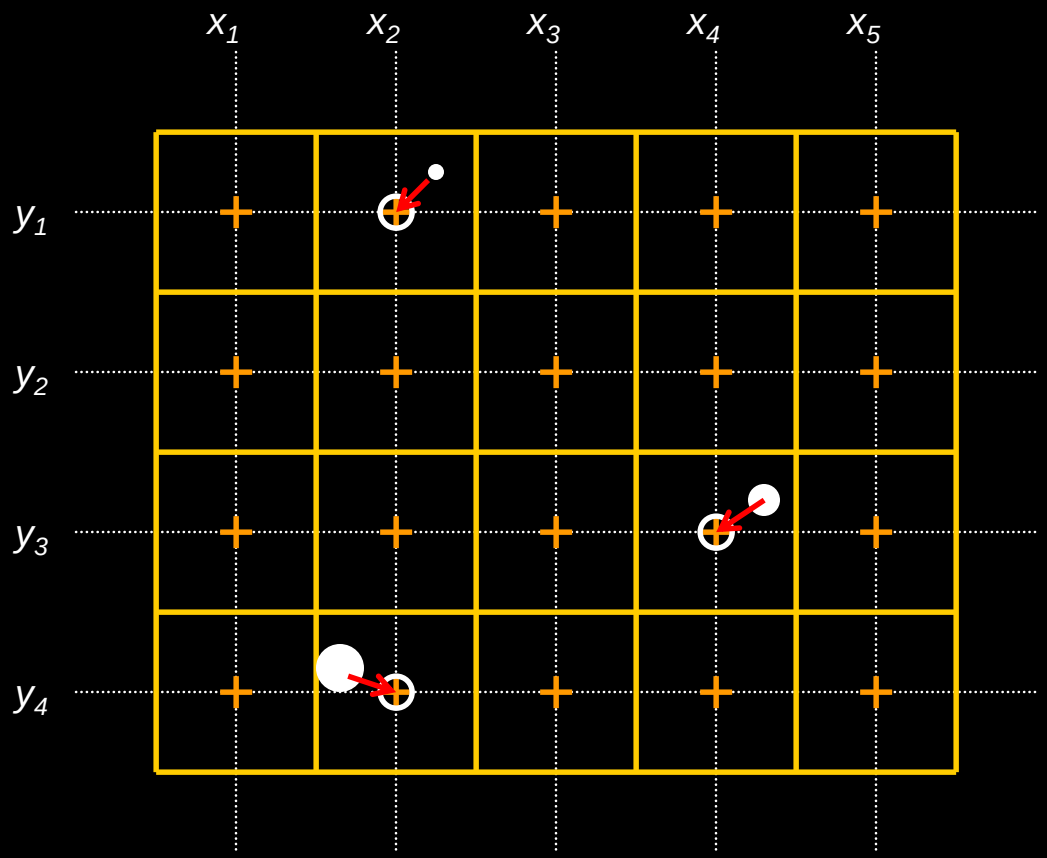
1. Particle Distribution → Density Function

A. Nearest Grid Point (NGP)
B. Cloud-in-Cell (CIC)
C. Higher Order Interpolations (e.g. TSC)

Particle-Mesh **(PM)**

Calculating Density Function: **NGP**

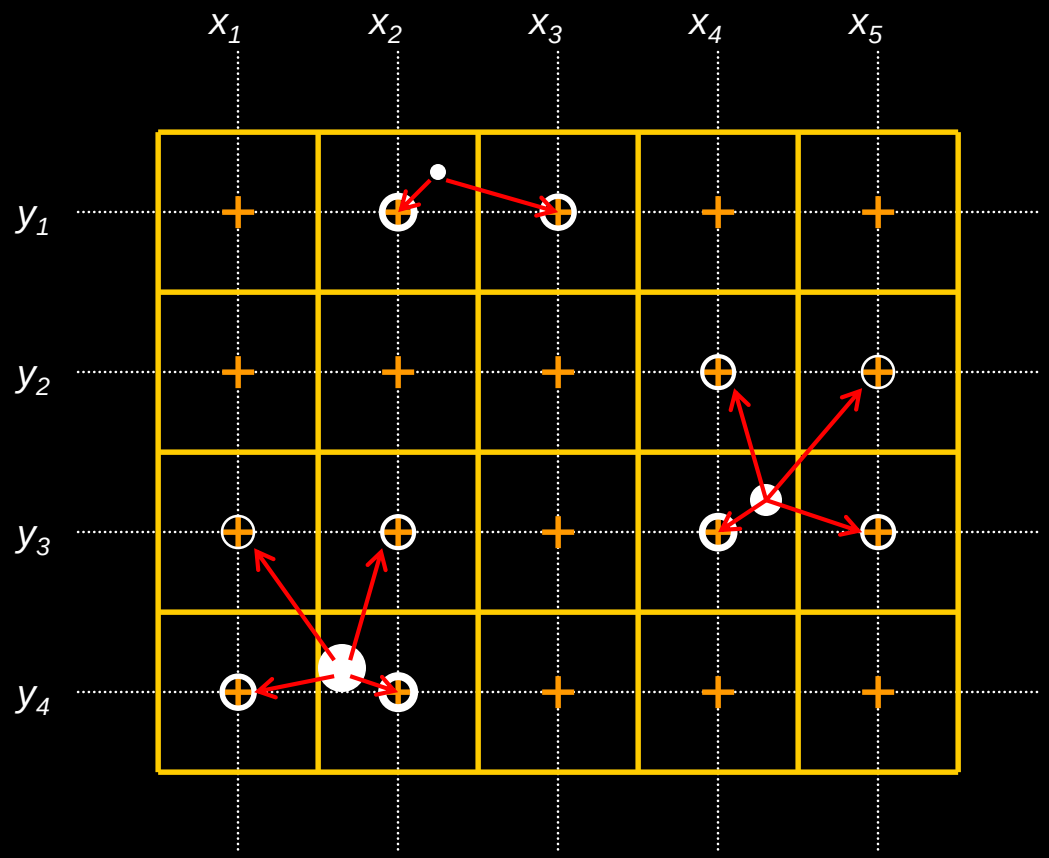
zero order interpolation



Particle-Mesh **(PM)**

Calculating Density Function: **CIC**

linear interpolation



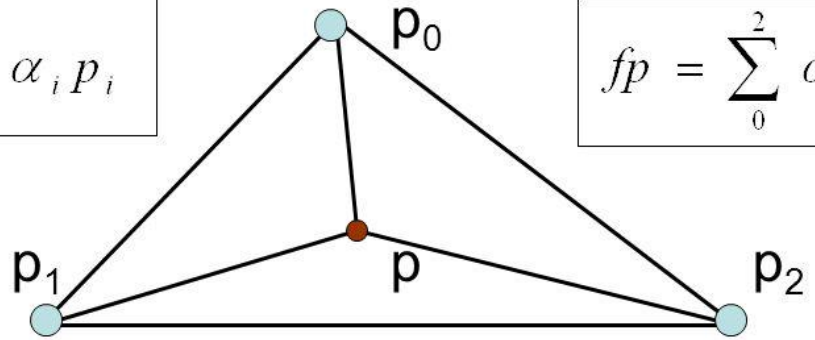
Particle-Mesh **(PM)**

Triangular-Shaped-Cloud

Linear interpolation over a triangle

$$p = \sum_{0}^{2} \alpha_i p_i$$

$$fp = \sum_{0}^{2} \alpha_i fp_i$$



For a triangle p_0, p_1, p_2, the Barycentric coordinates $\alpha = (\alpha_0\, \alpha_1\, \alpha_2)$ for point p,

$$\alpha = \left(\frac{area(p, p_1, p_2)}{area(p_0, p_1, p_2)}, \frac{area(p_0, p, p_2)}{area(p_0, p_1, p_2)}, \frac{area(p_0, p_1, p)}{area(p_0, p_1, p_2)}\right)$$

Particle-Mesh **(PM)**

2. Calculate Potential (mesh)

$$\Delta \Phi = 4 \pi G \rho(x)$$

Spectral method (FFT)

3. Calculate Forces (mesh→particle)

$$f_i = m_i \nabla \Phi , \quad i = 1...N$$

Particle-Mesh **(PM)**

+ Speed

$O(N + m \log m)$

+ Large scale phenomena

- Particle collisions
- Non-uniform distributions (<1 particle/cell)

Particle-Particle, Particle-Mesh **(P3M)**

Hybrid: (Long Range + Short Range)

1. Particle - Field

$$\Delta(\text{Particle,Partilce}) < 3\Delta(\text{Mesh})$$

2. Particle - Particle

$$O((N-n) + m \log m + n^2)$$

Particle-Particle, Particle-Mesh **(P3M)**

\+ Cosmology Simulations

\- Direct Summation can dominate

Improvements:

Adaptive Mesh

Wavelets

Particle – Mesh, Mesh **(PM2)**

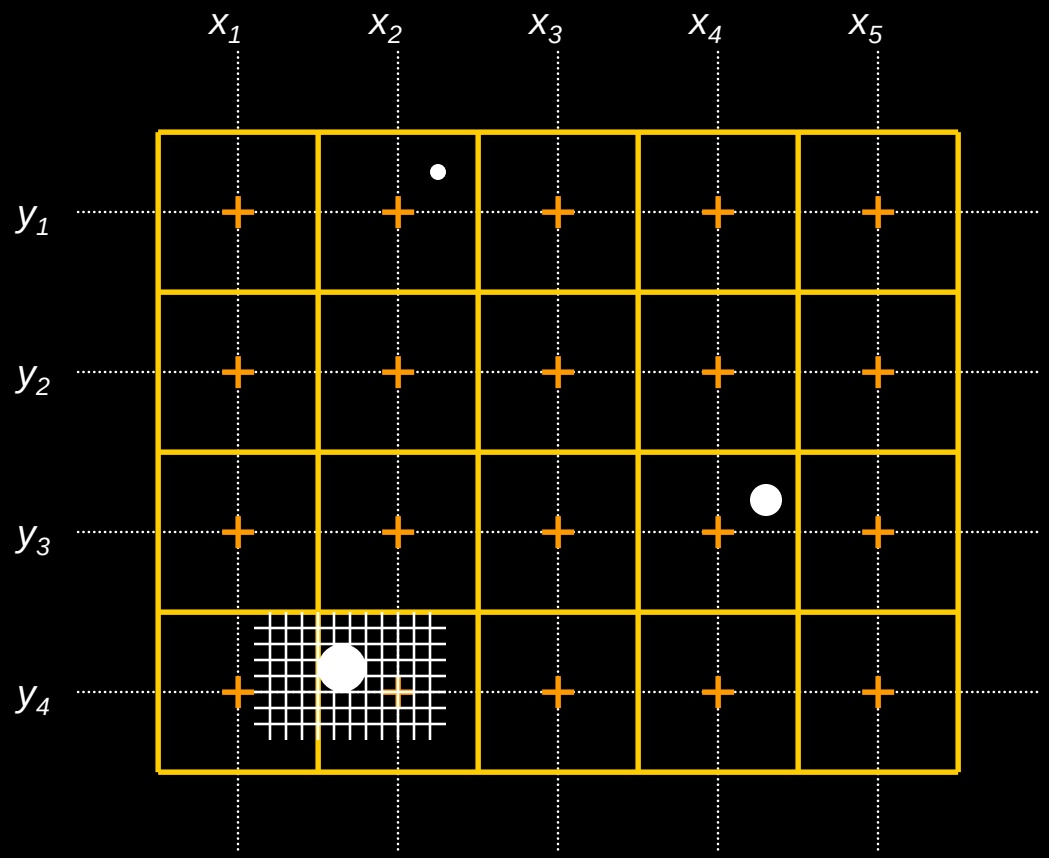
Particle + Mesh + Mesh

Particle
Global Mesh (coarse)
Particle Sub-Mesh (fine)

Long Distance Interaction (Global Mesh Field)
Short Distance Interaction (Particle Mesh)

Particle – Mesh, Mesh **(PM2)**

Particel – Mesh – Mesh



Particle – Mesh, Mesh **(PM2)**

$O(N + M \log M + \alpha\, m \log m)$

\+ Particle Collisions

\- Small Volumes

Max size for effect (galaxy) then PM

Nested Grid Particle – Mesh **(NGPM)**

Particle, Nested Mesh

Global Mesh, mesh1,mesh2,…

$\Delta M > \Delta m_1 > \Delta m_2 \ldots$

Smaller distance

Higher resolution (Forces)

Higher resolution (Mass)

Nested Grid Particle – Mesh **(NGPM)**

Particle mass → Grid density

Grid → Sub grid (Cloud-in-Cell interp.)

\+ Improved Particle Collisions

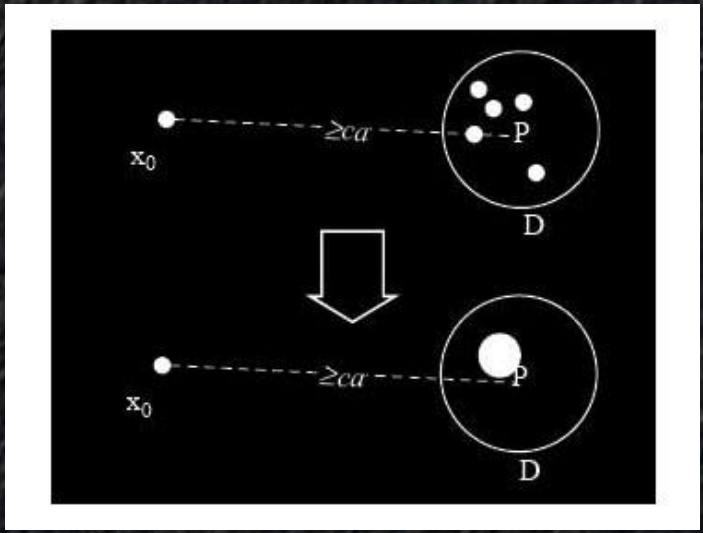
\- More Complex, Memory

Tree Code (TC)

Hierarchical Tree – Gridless

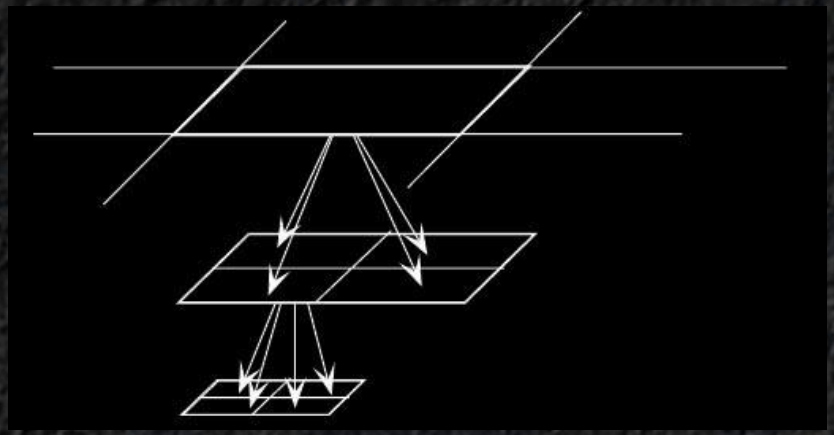
Interactions: Particle-Tree

Short range / Long range



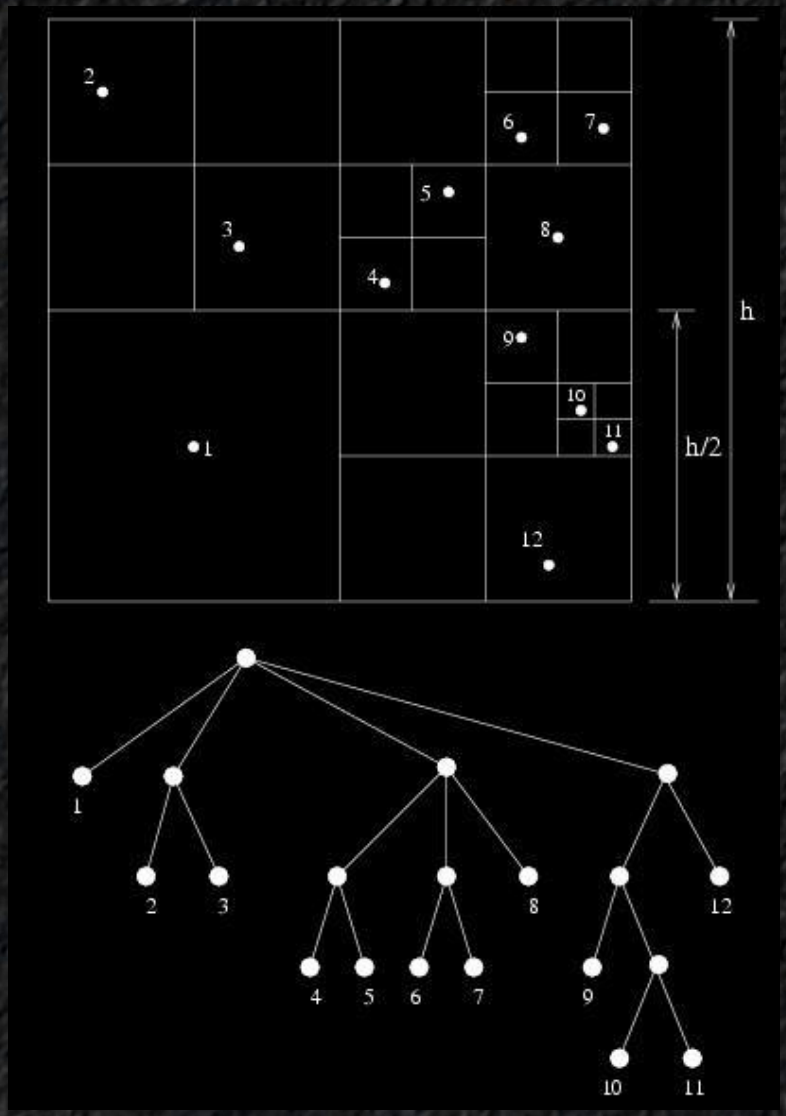
Tree Code **(TC)**

Hierarchy Tree:



Quadratee Partition

Barnes-Hut Tree



Tree Code **(TC)**

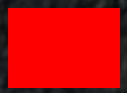
Different Hierarchic Trees

(Top up, Top down)

Barnes-Hut – Quadratees (corners!)

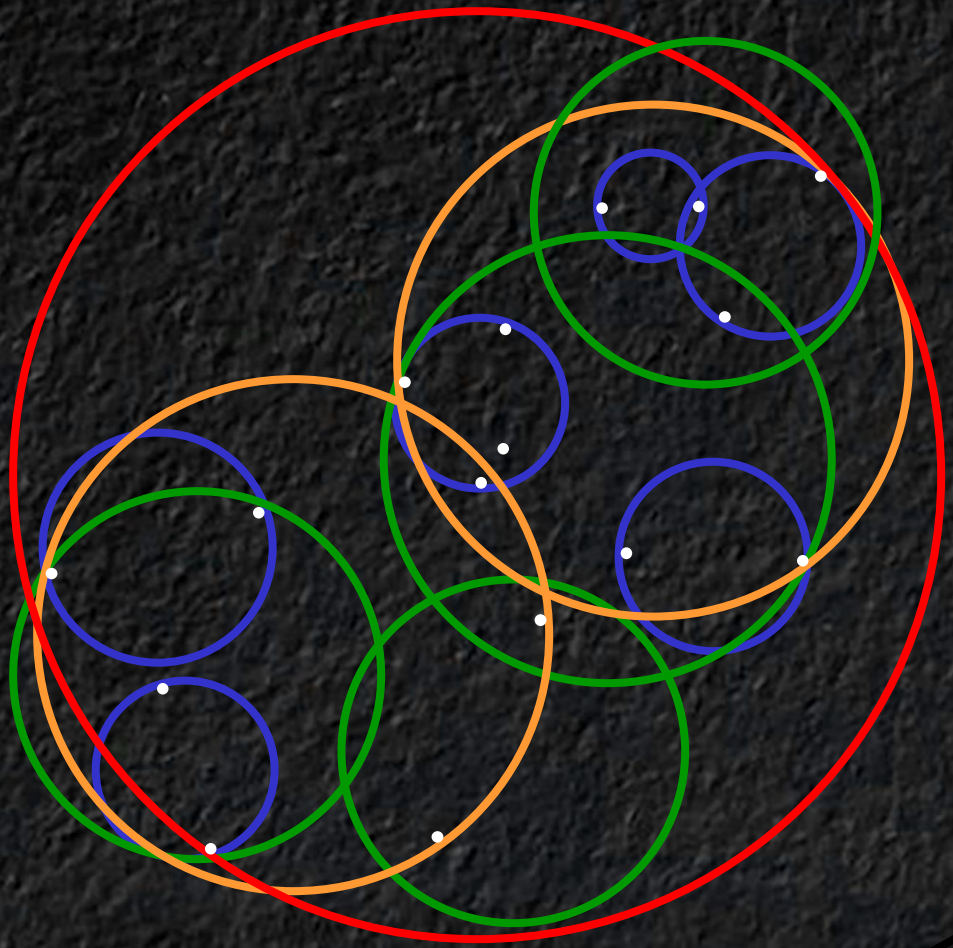
Ball Tree – Ball Tree

Ball Tree root node

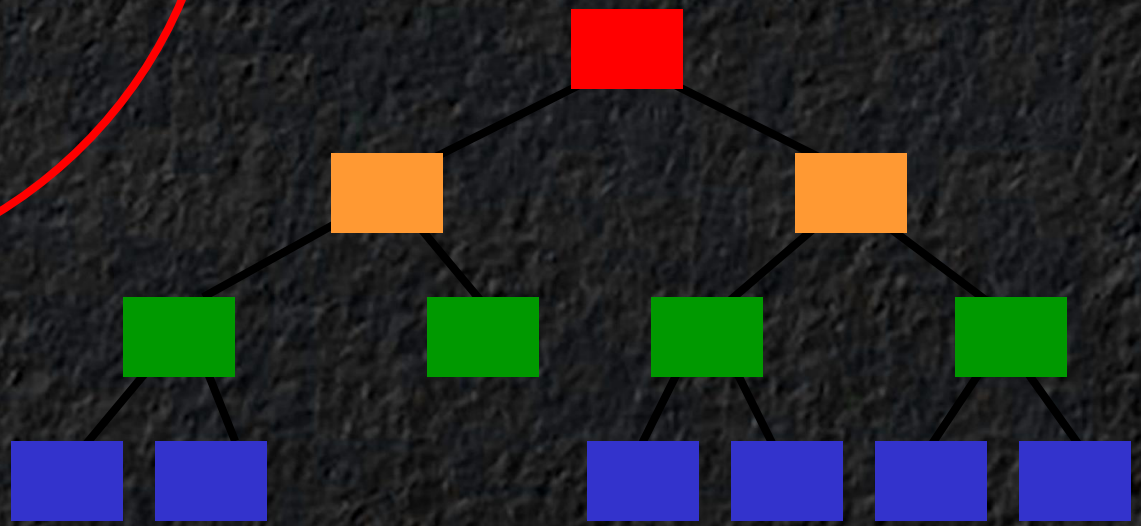


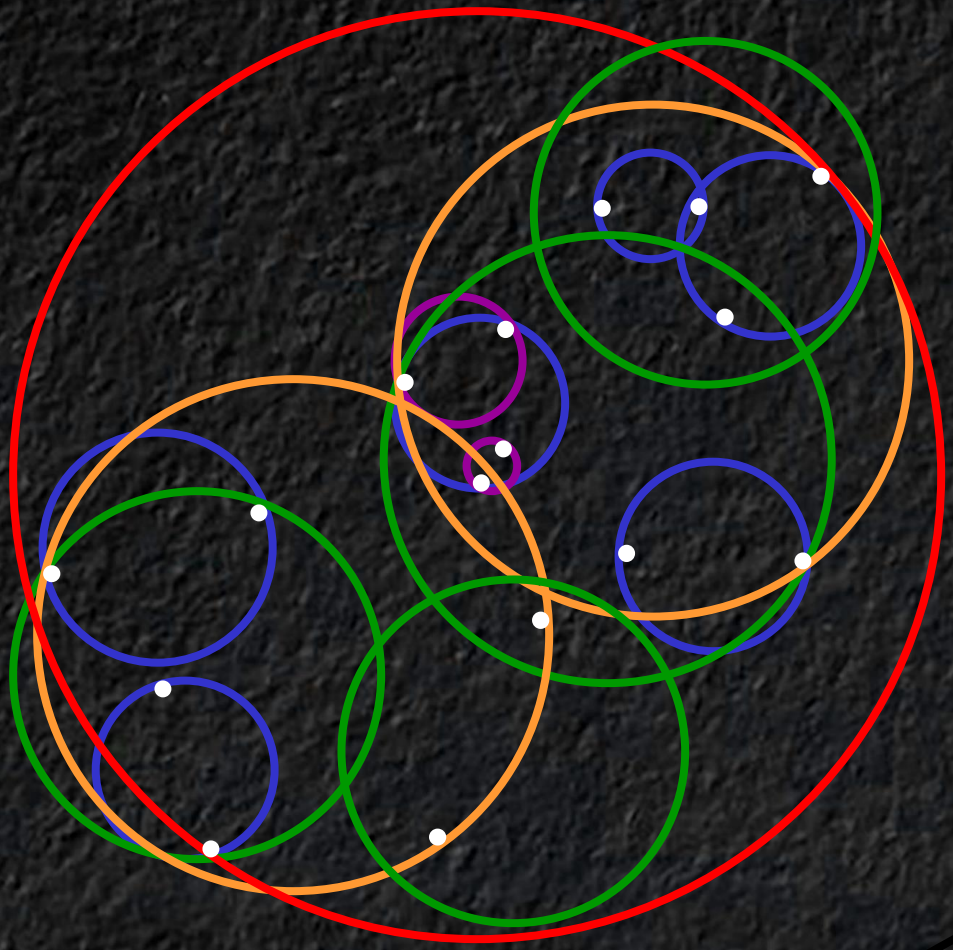
Ball Tree

Ball Tree

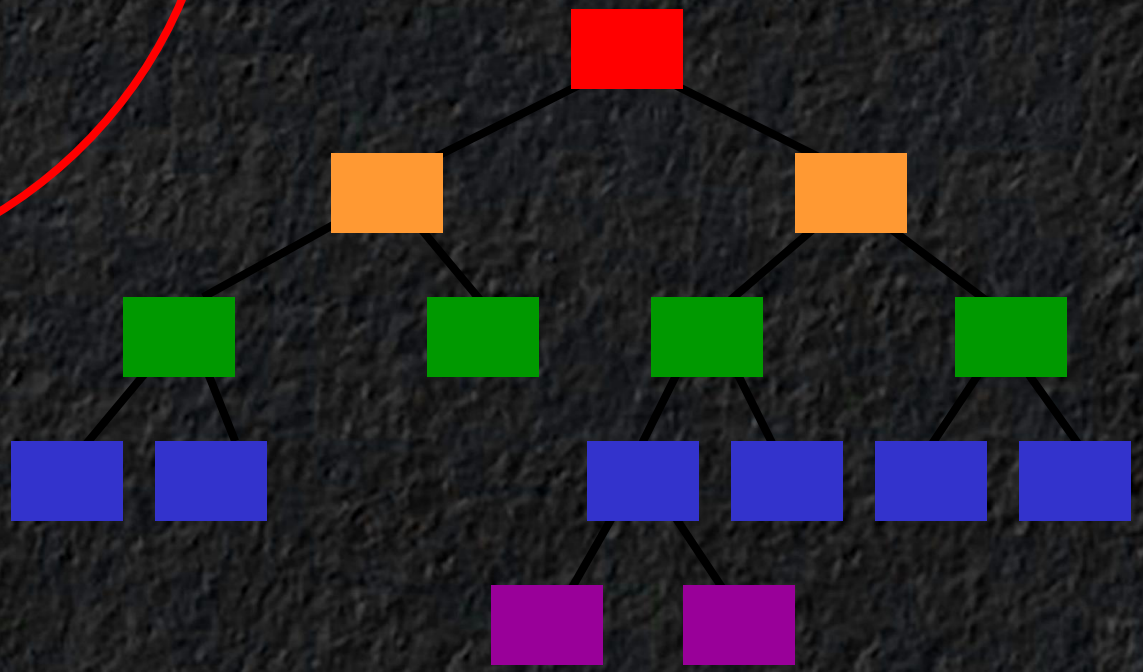


Ball Tree





Ball Tree



Tree Code **(TC)**

+ Flexible (Gridless)
+ $O\,(N \log N\,)$

- Memory (Store Trees)
- Accuracy (0-th order FFM)

Cell-Cell, Fast Multipole Method **(FMM)**

Particle / Tree code Hierarchy / Potential Field

Expand Potential Field into Multipoles

Monoples – R_1

Dipoles – R_2

Quadrupoles – R_3

$$R_1 > R_2 > R_3$$

Cell-Cell, Fast Multipole Method **(FMM)**

+ Faster then Barnes Hut (given accuracy)
+ O(N) ?
+ Better on Charged Particles

- Collisional Systems

end

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