ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი

ლექცია 10

Grids

Static Grids:

Uniform Grids;

- Cartesian
- Curvilinear

Non-Uniform Grids;

Irregular Grids;

Dynamic Grids:

AMR (Adaptive Mesh Refinement)

Grids

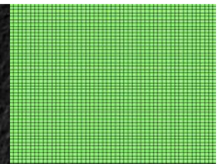
Grid Geometry Dependences

- Form of the Equations
- Numerical Differentiation
- Boundary Conditions

Why?

- CPU time
- Memory
- Accuracy

Uniform Grid



Numerical Differentiation

Forward

y'(x) = [y(x) – y(x-h)]/h

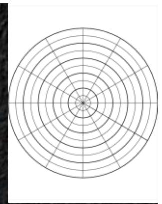
Backward

y'(x) = [y(x+h) – y(x)]/h

Higher order derivative

$$\frac{f(x+h) - f(x-h)}{2h}$$

Uniform: Polar



Curvilinear coordinates

1.) Euler Equation: additional terms

$$\frac{\partial V_r}{\partial t} + (\mathbf{V}\cdot\nabla)V_r - \frac{V_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r},$$

$$\frac{\partial V_\phi}{\partial t} + (\mathbf{V}\cdot\nabla)V_\phi + \frac{V_r V_\phi}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \phi},$$

$$\frac{\partial V_z}{\partial t} + (\mathbf{V}\cdot\nabla)V_z = -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial \Phi}{\partial z}.$$

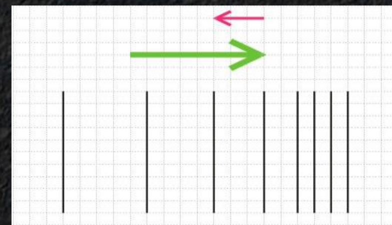
2.) Curvilinear Derivatives

$$(\mathbf{V}\cdot\nabla) \equiv V_r\frac{\partial}{\partial r} + \frac{V_\phi}{r}\frac{\partial}{\partial \phi} + V_z\frac{\partial}{\partial z}.$$

Non-Uniform Grids

Numerical Derivative:

$$\frac{f(x+h_2) - f(x-h_1)}{h_1 + h_2}.$$



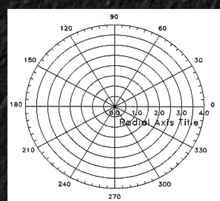
Stretching factor:
limited

Reflected waves from inhomogeneous grid: >15-20%

ასტროფიზიკის და პლაზმის ფიზიკის ამოცანების მოდელირება 2. ალ. თევზაძე (2016)

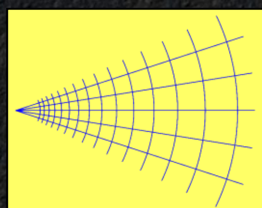
Non-Uniform Curvilinear

ϕ/r assymmetry



Radial stretching

$\Delta r_i = r_i \Delta\phi$



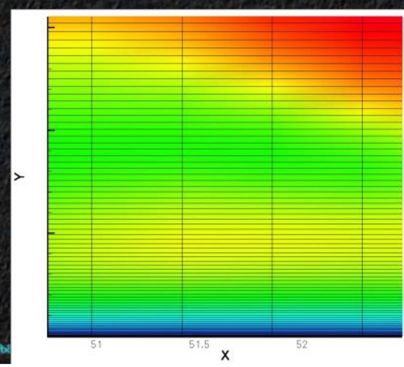
quasi-rectangular grid

ასტროფიზიკის და პლაზმის ფიზიკის ამოცანების მოდელირება 2. ალ. თევზაძე (2016)

Chebyshev Grid

Chebyshev polynomials

Spectral Method on non-uniform grids

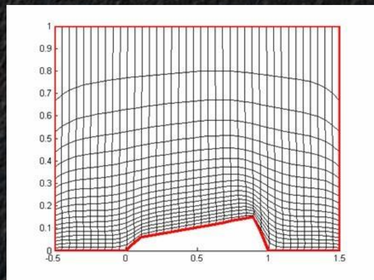


ასტროფიზიკის და ... ალ. თევზაძე (2016)

Irregular Grids

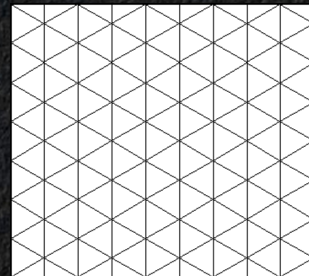
Grid Generation:
Problem-specific grid geometry

non-regular grids



Triangular Grids

Higher resolution (Reynolds number) for the similar grid points compared with Cartesian grid;

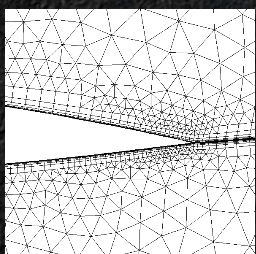


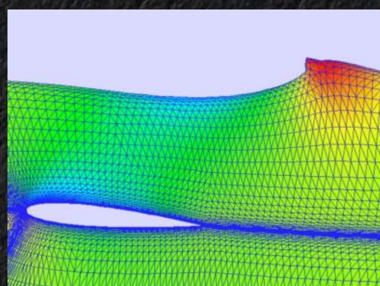
\+ Finite Volume

ასტროფიზიკის და პლაზმის ფიზიკის ამოცანების მოდელირება 2. ალ. თევზაძე (2016)

Complex boundaries

varying cell geometry / triangular
(industry)





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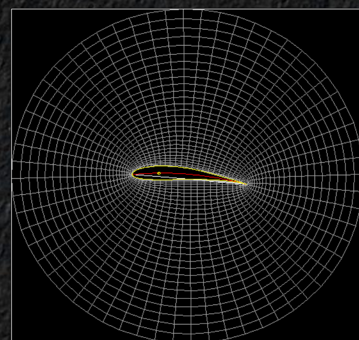
Irregular polar non-uniform



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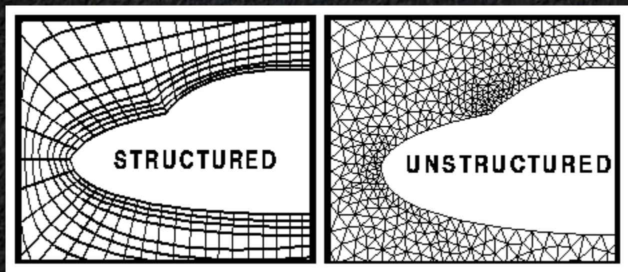
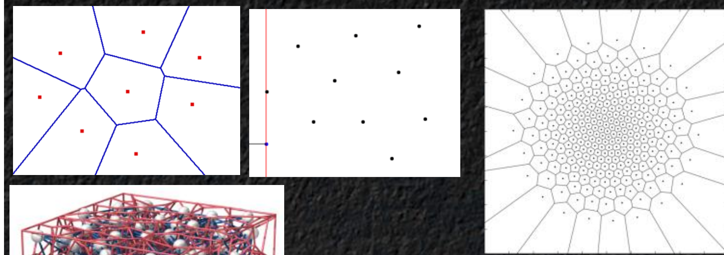
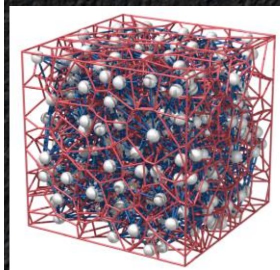
Unstructured grids

Irregular vs Unstructured grid



Voronoi Tesselation





Finite Volume – Cosmology sim
(AREPO);

Radiative transfer simulations
Camps et al. 2013

AMR

Adaptive Mesh Refinement:

Multi-scale problems
Scale 1: ***L1 ~ 0.01 m***
Scale 2: ***L2 ~ 100 m***

Δx ~ 0.001m
L ~ 1000m
N ~ 10^6

2D: ***N^2 ~ 10^12***
3D: ***N^3 ~ 10^18***

AMR

Dynamic mesh geometry:

Adaptation to problem

- Magnetic reconnection
- Self gravity
- Multiscale phenomena

Adaptive Mesh Refinement

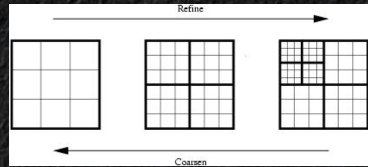
- Set refinement levels;

- Start with a course grid;
- Identify problem regions;
- Overimpose finer sub-grid;

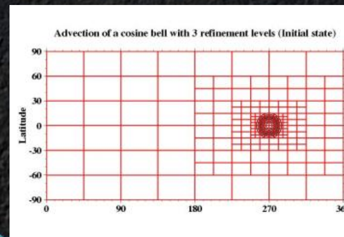
Save memory;

Save CPU;

AMR



Refinement levels



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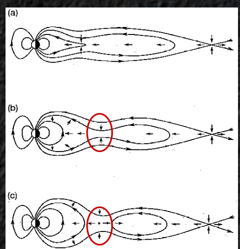
AMR problems

Reconnection



Gravitational clustering

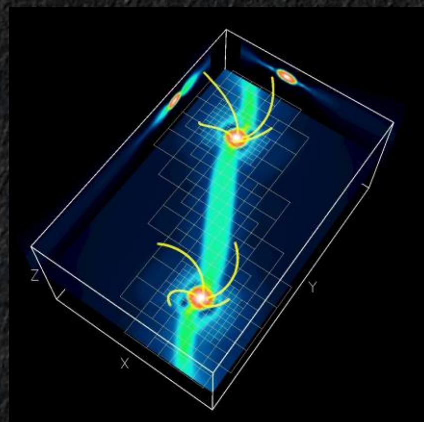
ასტროფიზიკის და პლაზმის ფიზიკის ამოცანების მოდელირება 2, ალ. თევზაძე (2016)

AMR

Star formation:

Density structure of a barotropic collapse with magnetic field, computed with NIRVANA3. adaptive mesh refinement and self-gravitation.

U. Ziegler (2005)



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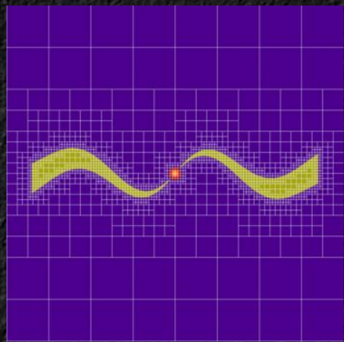
AMR in action



Summary

Static Cartesian Grid (simple, fast)

- Polar, Spherical (rotation, axial symmetry)
- Non-Uniform (increasing resolution, static setup)
- Chebishev (Boundary effects, spectral)
- AMR (Multiscale problems)

Direct comparison:

More Complex, More CPU, Less Memory

Performance = Balance (CPU,Memory)

end

www.tevza.org/home/course/modelling-II_2016/