Problem 4.32. (a) Find the included mass function $\mathfrak{M}(r)$ and the total mass M of a spherical star of radius R whose mass density (kg/m^3) varies with radius r as $\rho(r) = \rho_c\,[1 - (r/R)]$. (b) Find the central pressure P_c of this star in terms of M and R. (c) Find the central temperature in terms of M and R. (d) Compare your results for (b) and (c) to the answers to Problem 31 and comment. Hint: use (2), (1), and (14). [Ans. $M = \pi\rho_c R^3/3$; $P_c = (5/4\pi)GM^2/R^4$; ~0.4 $(Gm_{av}/k)(M/R)$;–]

Problem 4.44. For a neutron star of $M = 1.4\ M_\odot$ and $R_\odot = 10^4$ m, adopt the linearly-decreasing-density model of Prob. 32 (see answer to Prob. 32b). Let the star consist solely of free neutrons which, in degeneracy, behave as electrons. Assume the star is supported solely by completely degenerate, nonrelativistic neutron pressure. Find the following: (a) the approximate central pressure, (b) the neutron number density n_n and mass density ρ_c at the center, (c) the mass-radius relation and neutron-star radius, (d) the Fermi momentum of neutrons in the center of the star in energy units, $p_F c$, with comparison to $m_n c^2 = 940$ MeV, (e) the kinetic energy E_k and speed factor $\beta = v/c$ of the most energetic neutrons at the center of the star; calculated relativistically (Chapter 7) with comment, (f) an approximate maximum neutron-star mass limited by the (degenerate) neutrons' becoming relativistic, and (g) an approximate upper mass limit by requiring the Schwarzschild radius to equal the neutron-star radius obtained in part (c). The neutron mass is 1.675×10^{-27} kg. [Ans. $\sim 10^{35}$ N/m^2; $\sim 10^{45}$ m^{-3}, $\sim 10^{18}$ kg/m^2; ~15 km; ~700 MeV; ~200 MeV, ~0.6; ~5 $M_\odot$; ~4 $M_\odot$]