3.5 *Photon gas*

Problem 3.51. (a) Show that the energy density $u(R)$ J/m^3 of a photon gas expanding adiabatically (with no loss or gain of heat) in a spherical mirrored room of radius R decreases as R^{-4}. Consider a photon of initial frequency ν_0 that crosses back and forth across the full diameter $2R$. Let the mirrored walls of the container move outward at speed $v \ll c$ relative to the center. Start by finding the frequency change of the photon due to a single reflection; consider the Doppler effects for an observer at rest on the receding wall and another at rest with respect to the center of the sphere. From this, find how the frequency changes with R and then $u(R)$. (b) How does the temperature of the radiation depend on R? This is one way of understanding the cooling of the cosmic microwave background. [Ans. –; R^{-1}]

3.6 *Degenerate electron gas*

Problem 3.61. (a) Justify the two equal signs in (54), the relativistic expressions for kinetic energy E_F. If necessary, refer to Chapter 7. (b) Demonstrate that, at $p_F \ll mc$, the expression (54) yields the classical result $E_F = p_F^2/2m$.

Problem 3.62. What is the approximate Fermi energy E_F for the electrons in a white dwarf star of radius $R = 10^{-2} R_\odot$ and $M = 1 M_\odot$, in units of MeV? Assume the star consists of heavy elements so that there is about one electron for every two nucleons. Use a whole-star (average) value for the electron density. Are the particles relativistic (i.e., is $E_F \gtrsim mc^2$)? Hint: first, find p_F. [Ans. ~0.2 MeV]

Problem 3.63. Derive, with closed book, the nonrelativistic EOS (64) for a degenerate electron gas starting with the pressure expression (58).

Problem 3.64. Derive the relativistic EOS (69) starting with pressure-to-energy-density relation (66); include the steps skipped in the text.

Problem 3.65. Confirm that (64) and (69) have units of pressure (N/m^2).

Problem 3.66. The vertical dashed line in Figure 3.7 at $\log \rho = 9.58$ (kg/m^3) represents the place where the nonrelativistic and relativistic electron pressures, P_N and P_R, are equal. Find the value of μ_e adopted for this figure. What can you say about the composition of the gas? [Ans. ~2]