Visco-rotational shear instability of differentially rotating granular disks

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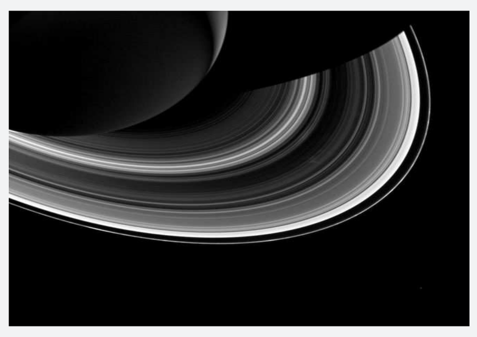
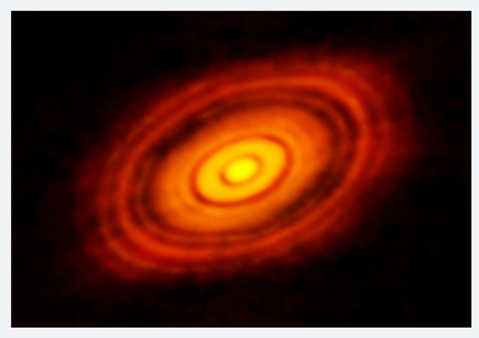
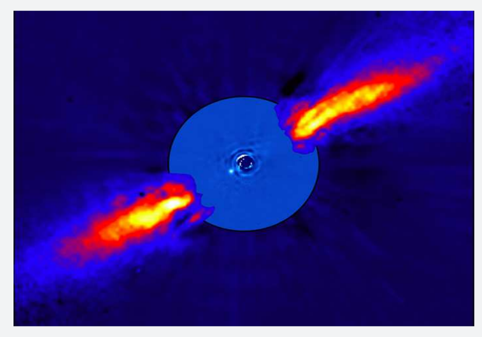
Abstract

We present the linear axisymmetric instability found in the incompressible differentially rotating flows with granular viscosity. The visco-rotational shear instability originates from the interplay of Keplerian differential rotation and shear rheology of granular flow.

Proposed physical model is based on the 3D local shearing sheet analysis of the linear stability of the Keplerian flows when the local constitutive equation of the granular flow can be used. Instability sets in granular flows, where the viscosity parameter grows faster than the square of the local shear rate at constant pressure. Dissipative stress of the granular flow leads to the growth of the vorticity perturbations with small radial and large vertical scales.

Visco-rotational shear instability can play a crucial role in the dynamics of dense planetary rings and granular flows in protoplanetary disks.

Granular flow

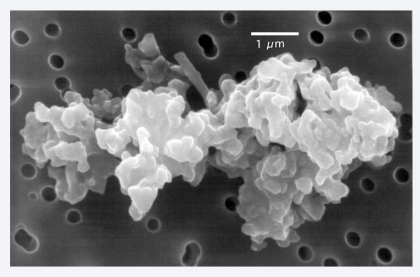
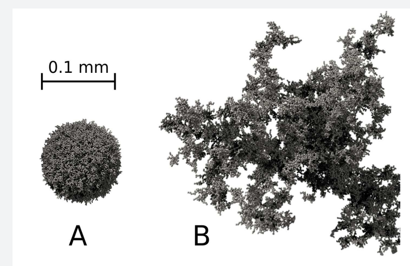




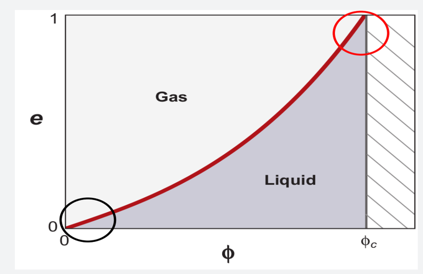
Flows of gas and dust rotating around central gravitation objects:

- Planetary and exoplanetary rings
- Protoplanetary disks
- Debris disks

Properties of granular matter:

- Mixture of gas and μm size solid particles;
- Highly porous particles: inelastic collisions;



Viscous Disk Model

Navier-Stokes equation:
$$\rho\left\{\frac{\partial}{\partial t}+V_k\frac{\partial}{\partial x_k}\right\}V_i=-\frac{\partial P}{\partial x_i}+\frac{\partial \Phi}{\partial x_i}+\frac{\partial \tau_{ik}}{\partial x_k},$$

Viscous stress tensor:
$$\tau_{ik}=\eta\dot{\gamma}_{ik}, \quad \dot{\gamma}_{ik}=\left(\frac{\partial V_k}{\partial x_i}+\frac{\partial V_i}{\partial x_k}\right)$$

Viscosity:
$$\eta=\eta(P,\xi), \quad \xi=\sqrt{\dot{\gamma}_{ik}\dot{\gamma}_{ik}/2}$$

Keplerian rotation:
$$\Omega_{kep}=\Omega_0\left(\frac{r}{r_0}\right)^{-q} \qquad (q=3/2)$$

Equilibrium radial profile of the viscosity parameter:
$$\frac{\partial \ln \bar{\eta}}{\partial \ln r}=q-3$$

Rayleigh stability criterion :
$$q<2$$

Non-Newtonian Granular Fluid

Pressure rheology parameter:
$$G_P\equiv\left(\frac{\partial \eta}{\partial P}\right)_\xi ,$$

Shear rheology parameter:
$$G_S\equiv\frac{1}{\rho}\left(\frac{\partial \eta}{\partial \xi}\right)_P$$

- **Viscose instability:** $\eta\propto\sigma^\beta, \quad \beta<0 \quad (G_P<0, \ G_S=0)$
- **Viscose overstability:** $\eta\propto\sigma^\beta, \quad \beta>2 \quad (G_P>0, \ G_S=0)$

Possible effects of varying viscosity:

- **Microscopic turbulence, eddy viscosity** (Shakura and Sunyaev 1973)
- **Granular rheology** (Jop et al. 2006; Forterre et al. 2008)

Local perturbed constitutive law: $\eta'=G_PP'+G_S\rho\xi'$

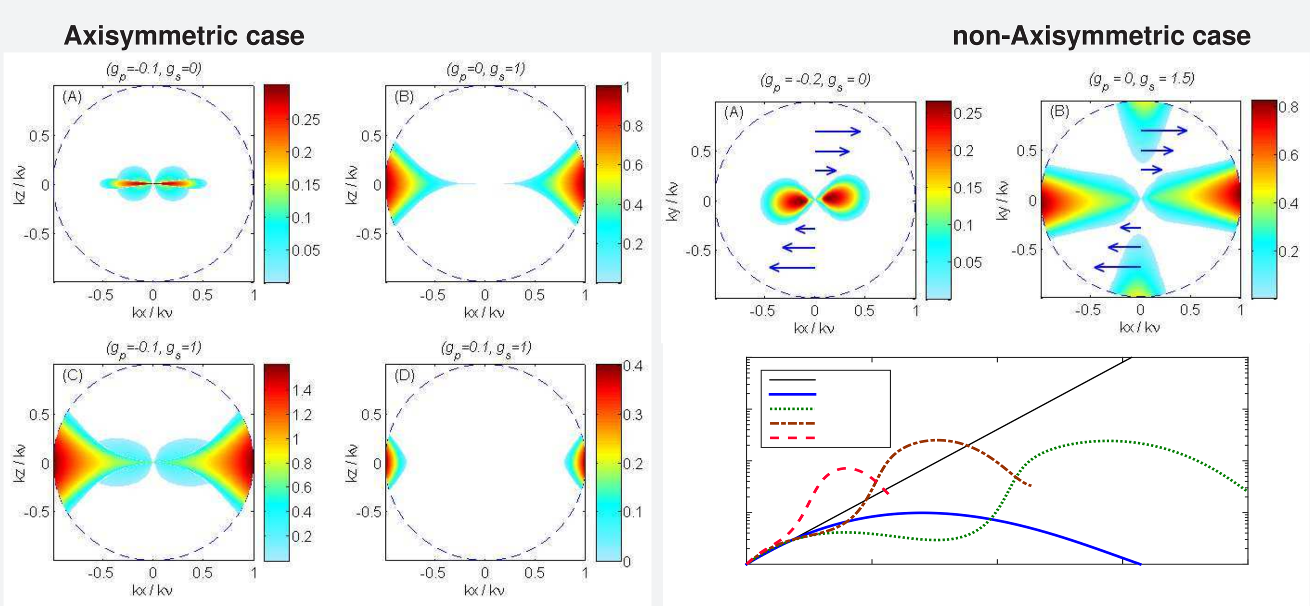
Generalized approach: $G_P\neq 0, G_S\neq 0$;

Rheology and Stability Analysis

$$\omega=\pm\left(\bar{\kappa}^2-W^2\right)^{1/2}+\mathrm{i}\left(W-\nu k^2\right),$$
$$W=\underbrace{\sigma_A}_{\text{Transient growth}}+\underbrace{\sigma_P}_{\text{Pressure reology}}+\underbrace{\sigma_S}_{\text{Shear reology}},$$

Visco-Rotational instability: $\bar{\kappa}^2>W^2, \ W>\nu k^2$ **for** $G_S>0$

Stability criterion:
$$\left(\frac{\partial \ln \eta}{\partial \ln \xi}\right)_P>2.$$



Summary

Visco-rotational shear instability indicates the possibility of destabilization of the narrow azimuthal ring (ribbon) structures. This axisymmetric instability becomes transient in nature for non-axisymmetric perturbations. It seems that this instability leads to growth of horizontal vorticity that can be described in vertically uniform $(k_z=0)$ shear rheology case $(G_P=0)$ by

$$\frac{\mathrm{d}}{\mathrm{d}t}\{\ln[\mathrm{curl}(\mathbf{u})_z]\}=qG_S\Omega_0\frac{(k_x(t)^2+k_y^2)^2}{k(t)^2}-\nu k(t)^2$$

The instability mechanism can be simply described using the pressure-vorticity balance: Anticyclonic vorticity perturbations to the Keplerian flow lead to local increase of the pressure. When this vorticity increase leads to the increase of the viscosity and corresponding accretion rate, pressure will increase even more, setting the linearly runaway process.